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SUPERCONDUCTING RESONANT CAVITIES

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SUPERCONDUCTING RESONANT CAVITIES

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1. INTRODUCTION

There is no doubt that the subject of superconducting resonant cavities is a fascinating field from both physical and engineering point of view.

The application of superconductivity to the world of resonant cavities has made achievable results unimaginable otherwise.

Independently of the special field of application, superconducting resonant circuits have superior performances compared to room-temperature circuits.

However the greatest resource of such devices stays not in the high quality of the results already obtained, but in all the potential applications and new ideas that must be still developed.

2. RESONANT CAVITIES

Everybody knows how to store or to transmit electromagnetic signals by means of electrical devices making use of linear passive components as resistors, inductances or capacitances.

At high frequencies, when the wave length of the signal starts to become less than 1 meter, normal circuits are hardly suitable for practical use and the only realistic way to store and to transmit electromagnetic radiation is to enclose it within metallic structures of size comparable to the considered wave length.

A resonant cavity is an energy storage device employed for the radiofrequency and microwave range. As such it is equivalent to a classical RLC resonant circuit.

At low frequencies the parallel-connected capacitor and inductance as sketched in fig 1 a, will resonate at a frequency $\omega_0 = 1/\sqrt{L\,C}$. To make this circuit resonating at higher frequencies, a possibility is to decrease L as much as possible. The inductance of fig. 1a can be decreased up to become a single wire. After that a further increase in ω_0 can be obtained disposing in parallel progressively many single-turns inductances (fig.1b).

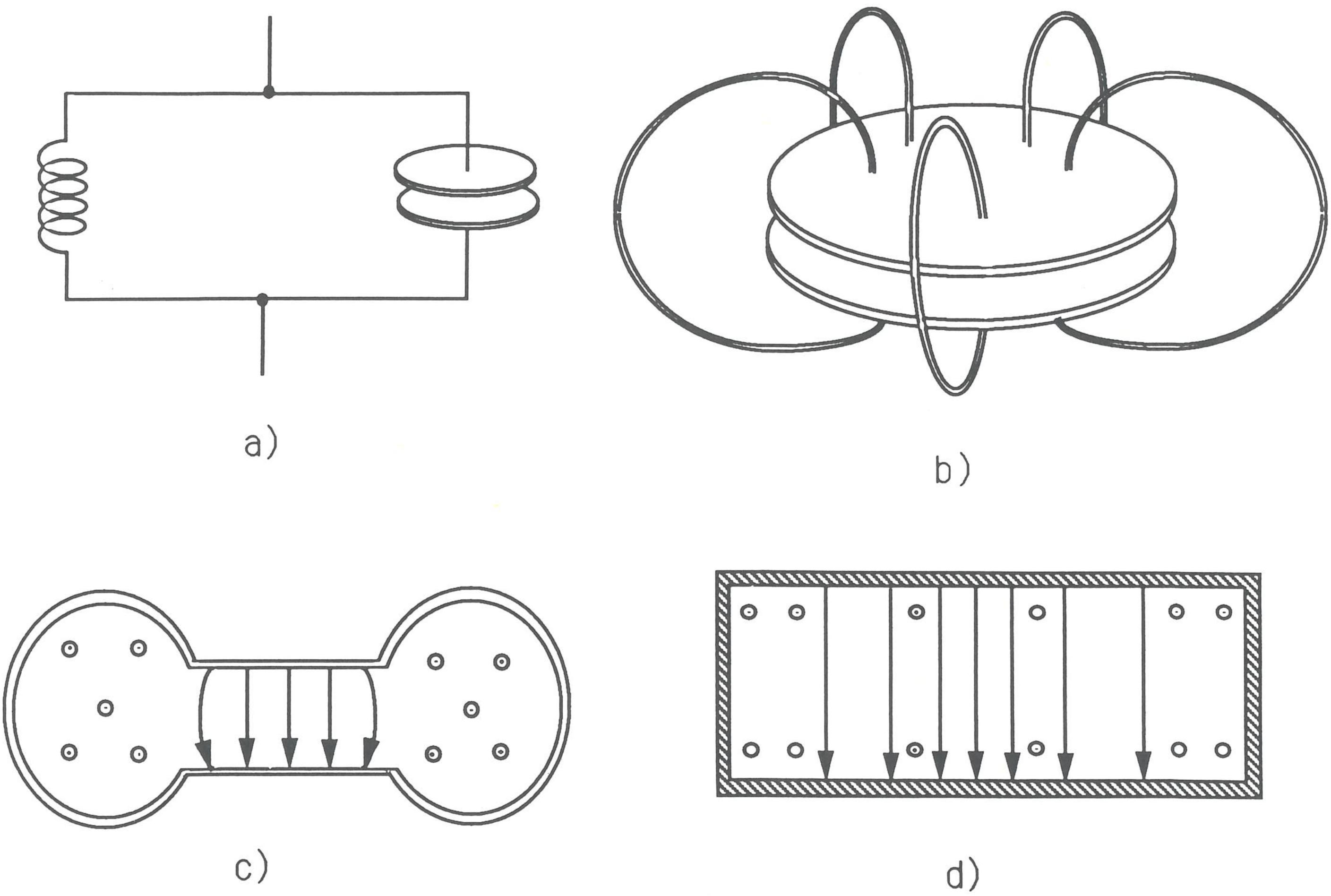

Fig. 1. *Metamorphosis of a LC circuit into a cylindrical pill-box resonant cavity (After R.P. Feyman[1]).*

The limiting case is the system of fig. 1c in which the two capacitor plates are fully enclosed by a metallic sheet that short circuits the capacitor. The next step in order to increase the frequency is then to decrease the size of the inductive part to the minimum. The result (fig. 1d) is the so called "pill-box" resonant cavity.

This example is, of course, not exactly rigourous, since capacitance and inductance become so strictly related that it must be examined the problem of distributed constants rather than of lumped constants. However it is significant since showing the logical evolution from lumped-circuit concepts to the idea of a resonant cavity.

It is immediate that in the configuration of fig. 1, the maximum voltage is obtained at the center of the two plates and that the electric field must fail to zero at the shorting walls. If we run over the logical path from the LC circuit to the pill-box resonator with the help of Maxwell equations after some approximations, it comes out that the electric field dependence along the radius ***r*** of the can is

$$E(r) = E_0 e^{i \omega t} J_0\left(\frac{\omega r}{c}\right)$$

where $E_0 e^{i \omega t}$ is the electrical field at the center of the condenser, and $J_0(x)$ is the Bessel function of zero order. The informations showed by the plot of $E(r)$, as displayed in fig.2, are several.

It is evident that different resonances take place, depending on which particular node of $E(r)$ is chosen for placing the short circuit wall. Indeed considering a pill-box of radius **a**, for each node n_i of $J_0(x)$, it exists a resonant frequency

$$f_i = \frac{n_i c}{2 \pi a}$$

These are not the only resonances one can excite inside the resonator; field arrangements different from the one of fig.1 are possible.

Fig. 3 indeed shows some simple fundamental resonant modes* for a cylindrical pill-box cavity.

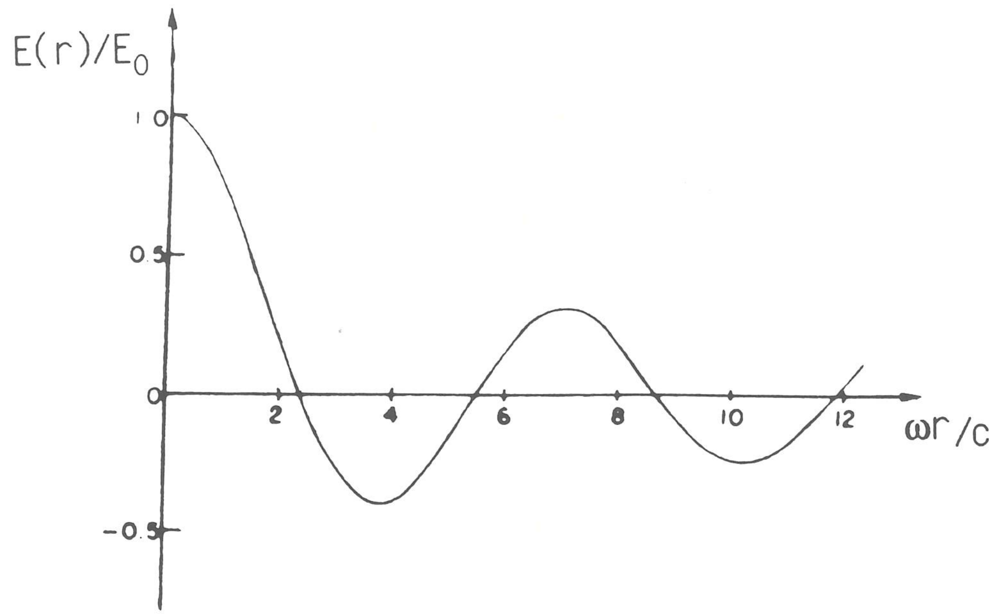

Fig. 2 *The electric field E plotted versus the quantity* $\left(\omega r/c\right)$.

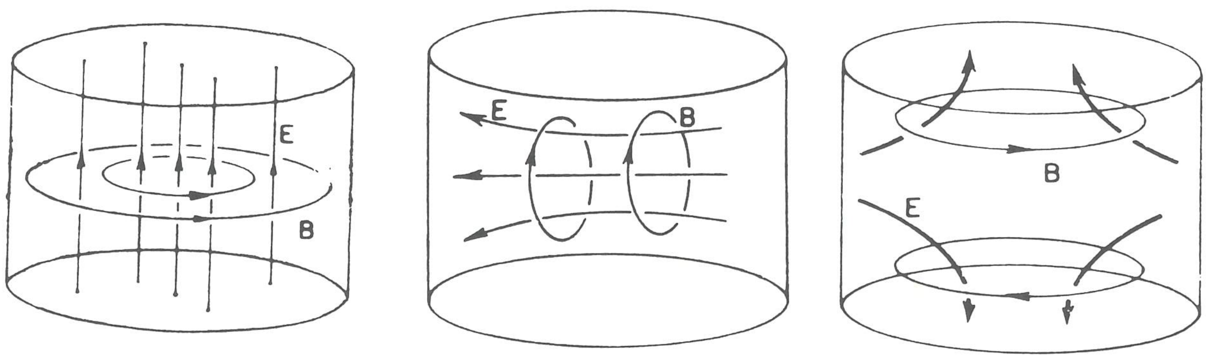

Fig. 3 *Different resonant modes of a cylindrical resonator. For simple cavities geometry the resonant modes are analytically obtainable solving the wave equation with the right boundary conditions.*

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* A mode is defined as an electromagnetic vibration of constant space distribution and sinusoidal time dependence.

Independently of which resonance is excited inside the cavity the basic principle is the same: the electromagnetic energy oscillates back and forth from entirely electric to entirely magnetic twice per cycle. At one instant the energy is all in electric form with accumulation of positive and negative charges on the top and bottom surfaces of the cavity. One-quarter cycle later the energy is all in magnetic form with currents flowing down the sidewalls.

If a cavity is left in free oscillation, without source, the field oscillation will get damped, due to the non perfect conductivity of the conductor or to the radiactive losses through resonator ports if present.

Then the negative change in time of the time-average energy *W* stored in the resonator will be equal to the dissipated power P

$$-\frac{dW}{dt} = P$$

If we define the the quality factor, or *Q* of the resonator as

$$Q = \omega_0 \frac{W}{P}$$

the energy initially stored into the cavity will exponentially decay with a damping time proportional to *Q*

$$W(t) = W_0 e^{-\frac{\omega_0}{Q} t}$$

The Q-factor is the most important parameter for a resonator. It specifies not only the energy leakage from cavity, but also it indicates its frequency selectivity (fig. 4). Indeed if $\Delta\omega$ is the bandwidth of the frequency response of the resonator it holds the relation

$$Q = \frac{\Delta\omega}{\omega_0}$$

For the most part of resonant cavities applications the ultimate Q is desired. This is one of the main needs for using superconductors in place of normal metals.

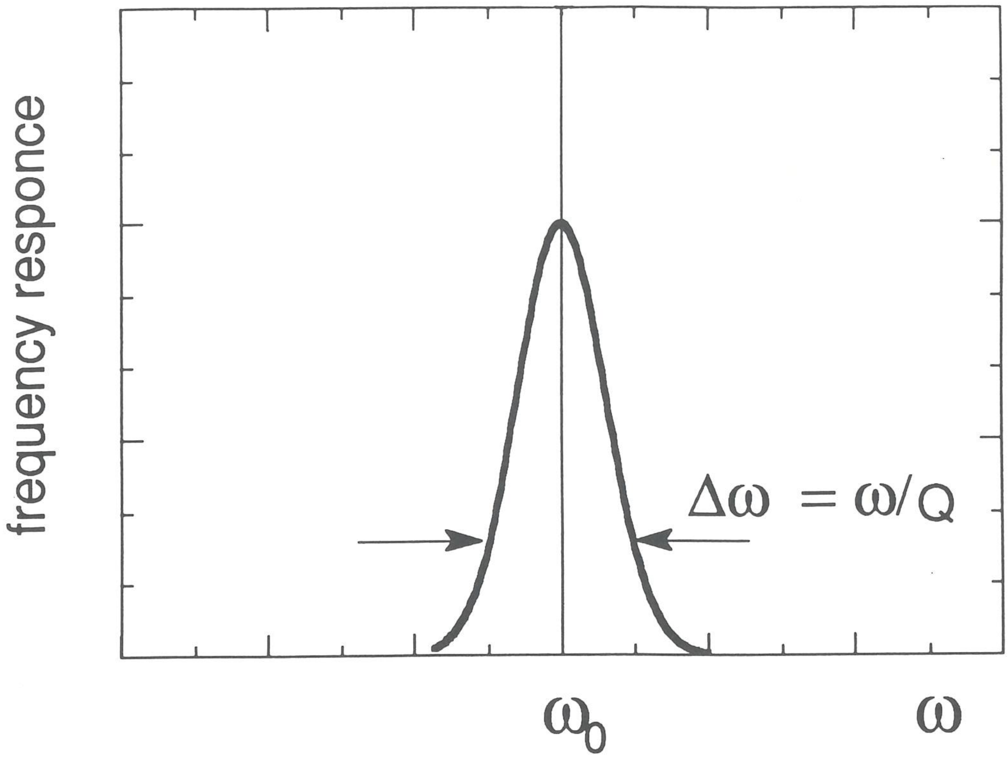

Fig.4 *The frequency response versus frequency. For normal RLC circuits Q-values of several hundreds can be easily achieved. By means of resonant cavities made of normal metals Q-values of* 10^5 *are obtainable. Q-values of* 10^{10} *can be reached only for superconducting resonators.*

3. THE SURFACE RESISTANCE

The resonator of fig. 1c has been derived from an ideal LC circuit containing no resistors. The pill-box walls were supposed to be done of a perfect conductor. That is clearly impossible for normal metal resonators, but it is also not realistic if the resonator walls are superconducting.

Indeed the perfect conductivity of a superconductor cooled below its critical temperature T_C is a phenomenon restricted to the zero frequency limit. At $\omega \neq 0$ a superconductor is far away from presenting no resistive losses.

If H is the magnetic field on the cavity surface, the power losses through the walls of a resonator may be described by the quality factor Q expressing it by means of a material dependent quantity R through the relation

$$Q = \mu \omega_0 \frac{\int_V H^2 \, dv}{\int_A R \, H^2 \, ds}$$

V is the volume enclosed by the resonator, A is the resonator surface. In the approximation of an uniform magnetic field on the surface such relation reduces to

$$Q = \frac{G}{R}$$

being G a geometrical factor.

R is the real part of a complex quantity, the surface impedance, that describes the material surface response to an electromagnetic field, and it is defined as

$$Z = \frac{E_t}{H_t} .$$

E_t and H_t are the tangent components respectively of the electric and magnetic fields to the metallic surface. That is absolutely equivalent to define Z as the ratio of the electric field on the metal surface to the current density integrated over the thickness of the metal

$$Z = \frac{E_t(0)}{\int_0^\infty j_t(x)\,dx}$$

As said before, **Z** is a complex quantity that can be written in the form

$$Z = R - i X$$

While the surface resistance *R* gives a measure of the dissipation processes of radiofrequency fields into the medium surface, the surface reactance X gives the phase shift of a wave upon reflection from the surface. So the quantities R and X can be determined from the change in amplitude and phase of the wave reflected from the metal surface.

3.1 The normal conducting case

When a rf electromagnetic field is oscillating in the cavity only the electrons within a thin surfacial layer δ (skin depth) of the resonator walls are interacting with the radiofrequency fields and the loss processes are so confined in such a layer.

According to the classical theory of skin effect for normal metals, in the normal regime, at a certain angular frequency ω, it holds

$$Z_n = \frac{1 - i}{\sigma_n \delta} = (1 - i)\frac{\rho_n}{\delta},$$

where $\sigma_n = 1/\rho_n$ is the *d.c.* conductivity at the working temperature.

Therefore we deduce that the quality factor of a normal conducting resonator is directly proportional to the square root of the d.c. conductivity.

Then one might expect that higher and higher Q-values are obtainable for cavities when building them for example from Copper of higher and higher purity, or even when cooling them down to low temperatures. Unfortunately due to the anomalous skin effect, the RF losses do not follow the $1/\sqrt{\sigma}$ law (fig. 5).

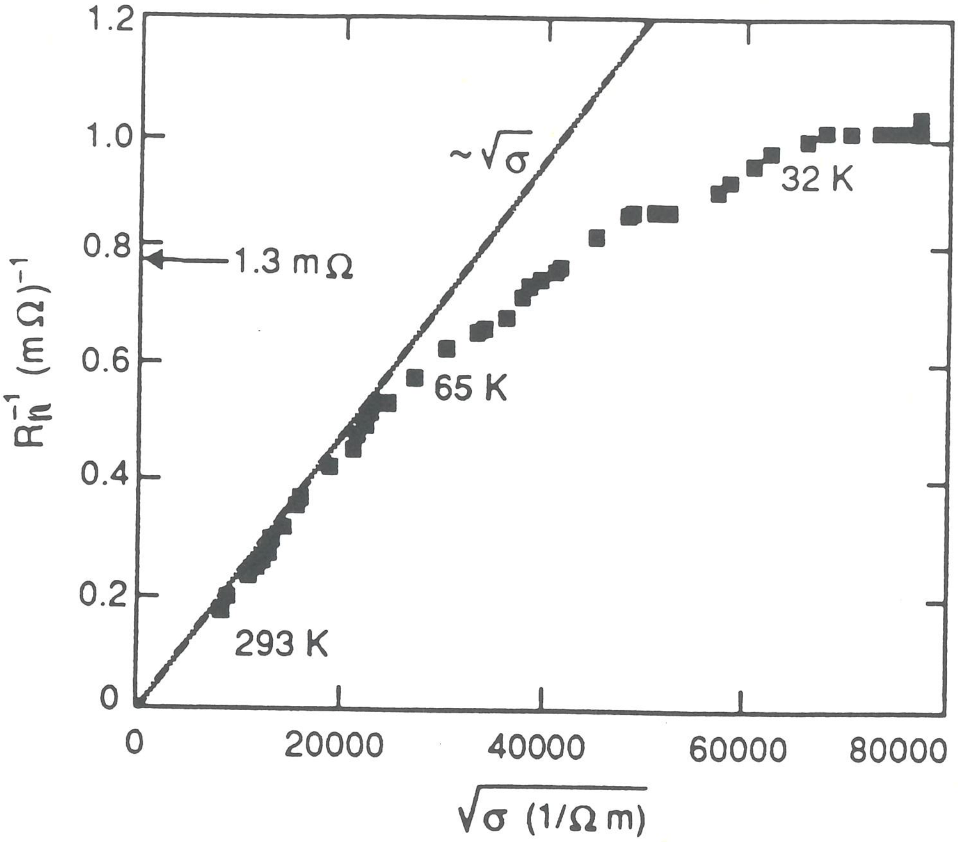

Fig. 5 *The anomalous skin effect for a 500 MHz OFHC Cu cavity. The experimental points deviate from the $\sqrt{\sigma}$ dependence (After W. Weingarten[2]).*

At sufficiently high frequencies indeed and when the material conductivity is increased up to the point that the mean free path ℓ becomes comparable to δ, it happens that the electric field varies significantly respect to ℓ.

Therefore the local relation between current and field $\mathbf{J} = \sigma\,\mathbf{E}$, obtained under the condition of a uniform field on the mean free path scale, is not valid any more and it needs to be changed by a non local relation.

In the limiting case of $\delta << \ell$, it can be argued that the only electrons travelling parallel to the surface participate to the conduction taking from the wave part of its energy. As displayed in fig. 6 the number of conduction electrons became approximately reduced of the factor ℓ/δ so the d.c. conductivity resulted reduced also of ℓ/δ.

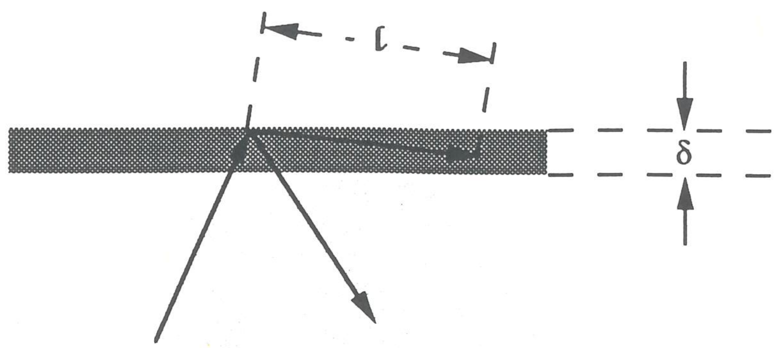

Fig. 6 *Only electrons having momentum confined within the skin depth are essential for conduction.*

Practically the existence of the anomalous skin effect prevents any further energy losses reduction instead expected by improving the metal purity or by lowering the operating temperature T.

This is the reason why normal Copper cavities used for example in accelerating rf structures operate at room temperature and not at liquid Nitrogen or liquid Helium temperatures. In the case of accelerating cavities cooled by liquid helium indeed the cryogenic losses would be even larger of the saving in rf power.

3.2 The superconducting case

What described for normal metals can be easily extended to superconductors[3], when in the formula for surface impedance a complex conductivity $\sigma_1 - i\,\sigma_2$ is introduced in place of the normal state conductivity σ_n.

The complex conductivity made its first apparition in the framework of the two-fluid model. It is surprising that the physics of a superconductor exposed to radiofrequency fields, from a descriptive point of view, is deeply understood by this simple phenomenological model. Even a quantitative analysis of the phenomenon can be performed with satisfactory agreement, only changing few details by the help of microscopical theory.

In a local field approximation, (i.e. for London superconductors), if an electrical field of the form $E_0\, e^{i\,\omega t}$ is assumed, the total current density $\vec{J}_T$ (the normal plus the superconducting component) corresponds to the field according to

$$\vec{J}_T = \vec{J}_n + \vec{J}_s = \left(\sigma_1 - i\,\sigma_2\right)\vec{E}$$

Hence, the complex conductivity is ruled out in terms of the fraction of normal electrons ${}^{n_n}/_{n}$ and that of superelectrons ${}^{n_s}/_{n}$.

$$\frac{\sigma_1}{\sigma_n} = \frac{n_n}{n}$$

$$\frac{\sigma_2}{\sigma_n} = \left(\frac{n_n}{n}\right)\omega\tau + \left(\frac{n_s}{n}\right)\frac{1}{\omega\tau}$$

τ is the momentum relaxation time and **n** is the total number of conduction electrons.

Of course the real part of conductivity is related to the dissipations associated to normal electrons within a London penetration depth λ while the imaginary part represents the inductive character of both normal and superelectrons.

Thus it is possible to sketch the equivalent circuit of the normalized admittance for a superconductor (fig. 7).

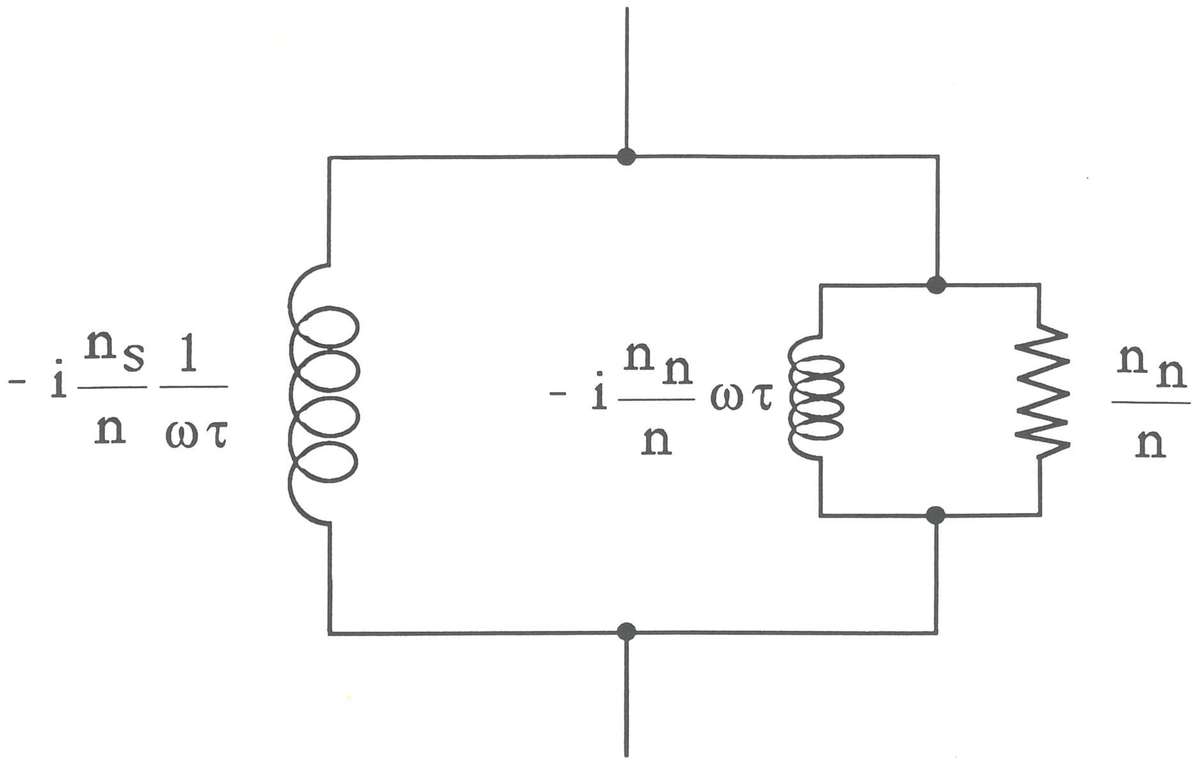

Fig. 7 *Equivalent circuit for the admittance of a unit cube of superconductor in the two-fluid model (After T. Van Duzer, L. Turner[36]).*

All we are looking for is written in this circuit:

- for $\omega = 0$, the resistance of normal electrons is short-circuited by the inductance of both normal and superelectrons.
- for $\omega \neq 0$, power losses start appearing since the inductance shunt effect at zero-frequency is less and less efficient the more ω get increased. The origin of RF losses becomes related to the presence of normal electrons.

The real and imaginary parts of the conductivity are plotted in fig. 8 as a function of the reduced temperature T_c/T .

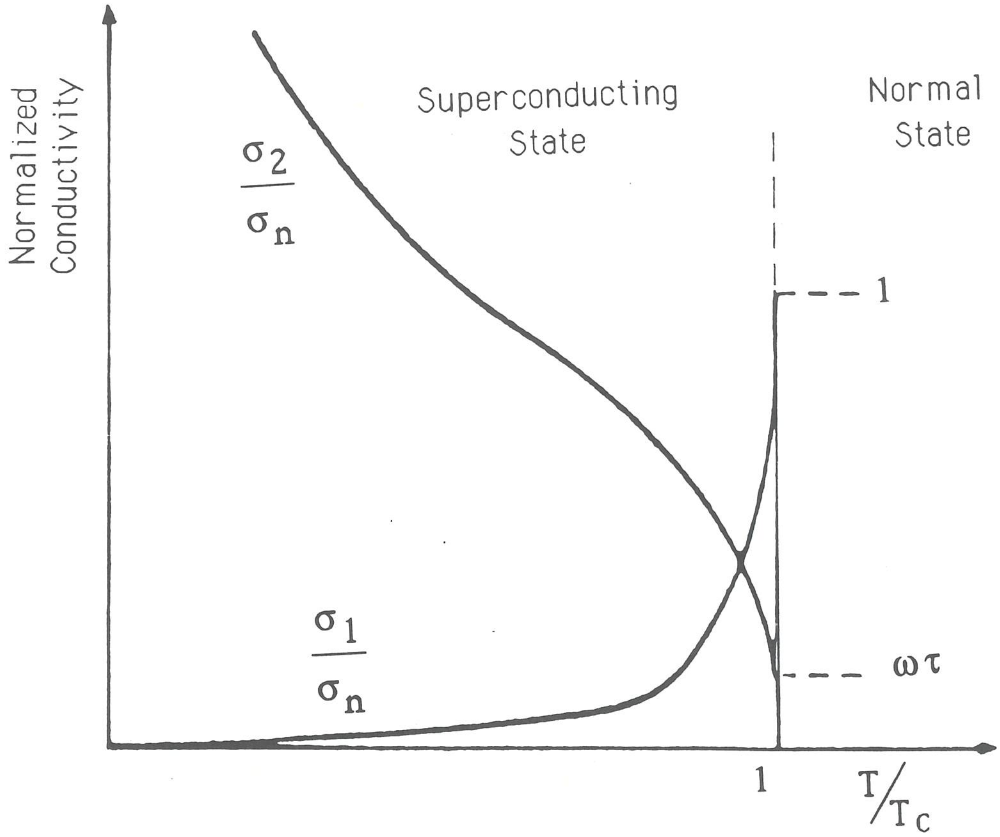

Fig. 8 *Real and imaginary part of conductivity versus temperature (After T. Van Duzer, L. Turner*[36]*).*

A magnetic field **H** oscillating parallel to the surface of the superconductor will induce an electric field ω **H** that furnish energy to normal electrons.

The power dissipated per single electron $\mathbf{P_e}$ will be so proportional to the square of the electrical field, $\mathbf{P_e} \propto (\omega \mathbf{H})^2$. Because we know from BCS theory that

$$n_n/n \propto \exp\left(-\frac{s}{2}\frac{T_c}{T}\right)$$

where s is the strong coupling factor, and T is the operating temperature.

The total power P_T will be $n_n P_e$ so the theoretical surface resistance will be proportional to ω^2 and to the exponential of $\left(-\ {}^{T_c}/_{T}\right)$.

A more rigourous treatment using the Mattis and Bardeen integral relations[4-6] for the complex conductivity* leads to the following result, valid in the dirty limit and for $T < (T_c\ /\ 2)$,

$$R_{BCS} \cong A\sqrt{\rho_n}\frac{e^{-\frac{\Delta}{KT_C}}}{\sqrt{s\ T_C}\,T\left(1+e^{-\frac{\Delta}{KT_C}}\right)^2}\,\omega^2 \ln\frac{\Delta}{h'\omega}$$

where $A \cong 6.00 \cdot 10^{-21}\left[\frac{\Omega K^3}{m\ s^4}\right]^{1/2}$

The term $\ln\left(\Delta/_{h'\omega}\right)$ represents a correction to the ω^2 law, due to the quasi-particle excitations entering more and more the anomalous limit when frequency becomes higher and higher.

Although limited to the dirty limit superconductors, such a formula clearly shows that low R_{BCS} values can be obtained for materials having low temperature normal state resistivity and high critical temperature.

In other words the BCS theory predicts that a good candidate for superconductive rf applications must be not only a good superconductor (high T_C), but also a good metal in the normal state (low ρ_n).

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* In the framework of the BCS theory extension at finite frequencies, Mattis and Bardeen arrive to the complex conductivity of a superconductor in the extremely anomalous limit. The MB expressions for conductivity were found also valid in the limit of impure superconductors by Nam[5].

3.3. The residual surface resistance.

The R_{BCS} dependence on ρ_n and T_c represents an immediate criterion for selecting, among superconductors that are the most favourable materials for superconducting cavities.

Unfortunately the validity of such a criterion is compromised by the fact that the BCS prediction of a vanishing surface resistance at **T= 0** is never verified in practice because of a temperature independent residual term R_{Res} generally of tenths of $n\Omega$ in the measured surface resistance (fig.9):

$$R_S = R_{BCS}(T) + R_{Res}$$

The term "residual" is an indication that the causes of those losses are often not clear. Indeed even if much is reported in literature about the possible origins of the non-BCS losses, they are far from being completely understood, mainly because both "physical phenomena" and "accidental mechanisms" (like dust, chemical residuals or surface defects on the cavity walls) contribute to the residual. In particular the "accidental mechanism" makes difficult to identify the physical causes from the experimental results that often are not highly reproducible even for the same cavity when cooled and measured several times.

In literature several kind of mechanisms trying to explain the residual surface losses have been proposed.

Here we list among the possible sources of residual losses those have been more investigated and discussed:

a) <u>Losses due to a non-ideal surface quality</u>[7]:

The non-controllable nature of this kind of losses makes them rather difficult to analyze. On the other hand it is universally recognized that the worst polished superconducting surfaces show the highest residual losses. The enhanced phonon generation taking place on real surfaces due to the unavoidable presence of surface irregularities is proposed in literature as a possible physical mechanism responsible for such type of dissipations.

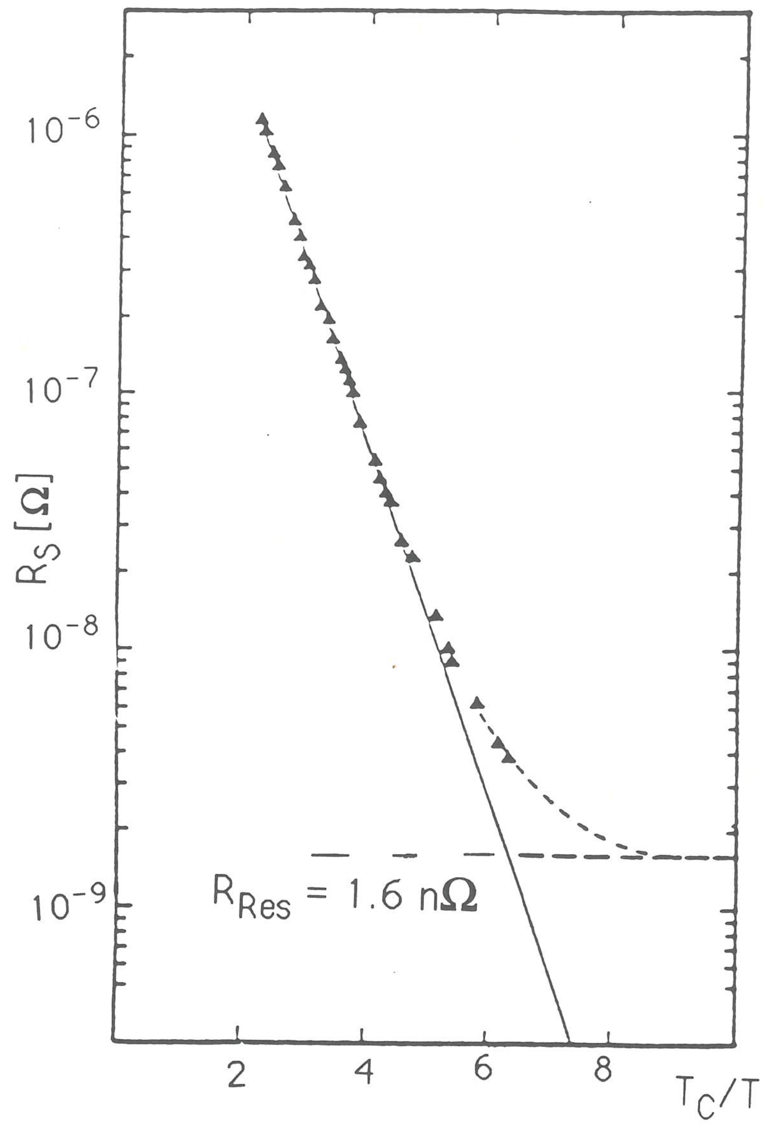

Fig. 9: *A classical picture for the residual surface resistance. The dashed line is the residual term, while the continuous line is the BCS predicted surface resistance.*

Inclusions of metallic foreigner particles into the superconductor within a London penetration depth λ from the surface can also be source of Joule losses. Depending on the thermal conductivity of the superconductor and on the contact thermal resistance between the normal particles and the superconductor, normal areas will dissipate more and more, as soon as power is injected into the cavity. If the thermal conductivity of the surrounding material is not high enough to remove the heat developed by normal islands to the liquid helium bath, the local temperature gets higher and higher. As a result the critical field gets lower and lower. All this can result into a sudden transition from the normal state to the superconducting state (quench) and can constitute a severe limitation to the achievement of high fields inside superconducting resonators.

b) Losses due to an Oxide layer.present on the superconducting surface[8-10]:

It must be kept in account that the only two superconductors most widely used for superconducting cavities applications are Lead and Niobium. Without doubt when high performance resonators are required, the Niobium choice becomes compulsory.

Respect to Lead indeed Niobium has a 2K higher critical temperature, and a critical field H_C at **0 K** of ~2000 Gauss against only ~800 Gauss of Lead.

Moreover, Niobium is superior to Lead even from the surface stability point of view: Lead has several stable oxides; Niobium has only two oxides Nb_2O_5 and NbO.

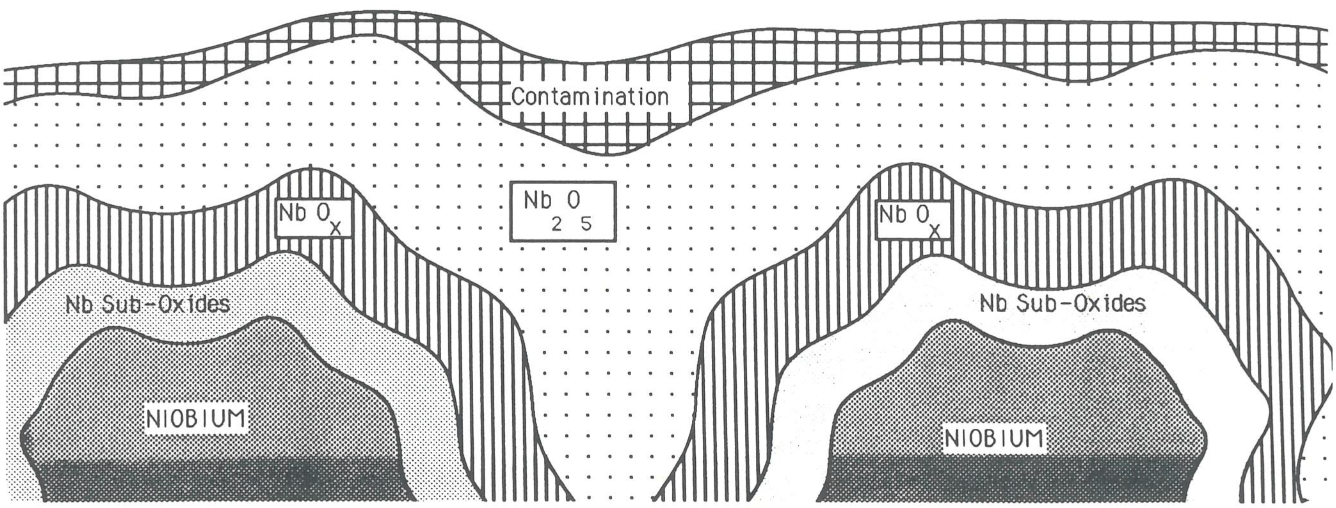

Fig 10 *A real Nb surface exposed to air at the room temperature.*

Due to the too small thickness of saturation, NbO plays a minor role in the mechanism of rf losses. Nb_2O_5 also is recognized to be not significant from the point of view of rf residual losses, since for example, as indicated by Halbritter[8)] only fractions of $n\Omega$ are expected in a 6 nm Nb_2O_5 layer without defects.

On the contrary a possible source for residual losses can be represented by the interface suboxide precipitates mainly ($NbO_{0.02}$) between Nb grains and NbO. An Oxygen content of only 1 at. % dissolved inside Niobium indeed reduces the critical temperature T_C by 10 %.

In fig. 5 it is sketched the Niobium - Oxygen system existing on a real Nb surface handled in air at 300 K[8)].

c) Losses due to the electromagnetic generation of acoustic phonons[10-14)]

If the action of transverse radiofrequency fields on the lattice ions are considered, the generation of transverse ultrasonic oscillations becomes important and the conversion efficiency of electromagnetic to acoustic energy rapidly increases in the GHz range at low temperatures when increasing the purity of the material.

d) Losses due to the trapping of magnetic flux[15-17)]:

Trapped flux losses due only to the background earth field have been universally recognized as a primary source of residual losses.

Because of the large demagnetization factor of the extended surface of a resonator, an incomplete Meissner-Ochsenfeld effect happens, favouring the trapping into the superconductor of any external field present during the cooling phase. The adoption of Helmholtz coils or high magnetic permeability screens around cryostats in fact sensitively improves the rf performances of superconducting cavities*.

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* In the framework of superconducting cavities for particle accelerators, three different families of resonators mainly exist: Lead electroplated Copper cavities, Bulk Niobium cavities and Niobium Sputtered cavities.

It must be remarked that while electroplated Lead and bulk Niobium are clearly sensitive to the trapped magnetic field, the Niobium sputtered solution seems to be unaffected by such a problem[18)].

Two possible mechanisms of dissipations, one statical and another one dynamical, are possible for the magnetic flux trapped into the walls of a superconducting cavity: the dissipation of the vortexes cores seen by the rf fields as normal areas and the flux flow resistance due to the vortexes dynamical flow.

Depending on a plethora of circumstances two different dependences for the residual resistance are mainly mentioned in literature:

$$R_n \left(\frac{H}{H_{C2}}\right) \qquad \text{or} \qquad R_n \left(\frac{H}{H_{C2}}\right)^{1/2}$$

being H the trapped magnetic field and Rn the material low temperature surface resistance in normal state.

e) <u>Losses due to the superconductor polycrystallinity</u>[19]:

For type II superconductors the rf and microwave surface impedance can be strongly limited by dissipative process determined by the intergranular coupling. Additional surface resistance can indeed arise due to the nature of grain boundaries behaving as weak links of Josephson junctions. Such mechanism has been firstly considered for high T_C superconductors[20], recently it has been investigated for Niobium[21].

f) <u>Losses due to the Hydrogen segregation</u>[22]:

A *nouvelle* kind of additional losses during last few years has been recently recognized to be, in particular for Niobium cavities, a potential enemy to fight. It is well-Known that Niobium, if electropolished or chemically treated by immersion in Hydrogen-rich etchant can be charged at least on surface by Hydrogen.

Hydrogen mobility in Niobium is known to be very high; moreover it is clear that if diffusion there is, it will happen through grain boundaries rather than through single grains. Hydrogen should first enter into Niobium interstitially at room temperature and then precipitate to form

Hydride islands randomly distributed inside Niobium, when cooled below **120 K**.

This hypothesis might explain the so-called "Q-disease" and the "100 K effect" for Niobium cavities, i. e. the deterioration of Q versus field depending on the particular chemical treatment or on the cooling rate especially observed after a long warm-up of the cavity at about **100 K**. The hypothesis of the hydrogen effect on the cavities performances is certainly interesting. However much work still must be done for arriving to a full confirmation and understanding of the real effects of Hydrogen.

g) General considerations about the residual losses:

As we shall see in the following it is not possible to give one formula predicting the residual losses, since their potential sources are too many and often too fragmentary informations exist about them.

Nevertheless there is experimental evidence that R_{Res} rises with R_n, that is proportional to the square root of ρ_n. Moreover if R_{Res} is the summation of several contributions of thoroughly different nature, it is not improbable that its frequency dependence is a linear combination of terms having different powers. Any way among the many (often contradictory) data in literature, there is a certain evidence[23-25] that the strongest contribution has approximately the form of $\omega^{2-\delta}$.

4. SUPERCONDUCTING CAVITIES FOR PARTICLE ACCELERATORS.

Without doubt the main application of superconducting cavities is the one of large particle accelerators. For such a technology it is a well-established concept that superconductive resonant rf structures make possible objectives otherwise impossible or costly prohibitive.

Since the first project of a superconducting accelerator started at Stanford[26] in 1962, much work has been done in this field. The improvement of RF superconducting technology due to the joined effort of universities, national laboratories and industries have strikingly enhanced the obtainable performances and consequently have pushed even farther the goal to reach. The energy levels achievable for future accelerators are indeed intimately dependent on the progress of RF-superconducting cavities.

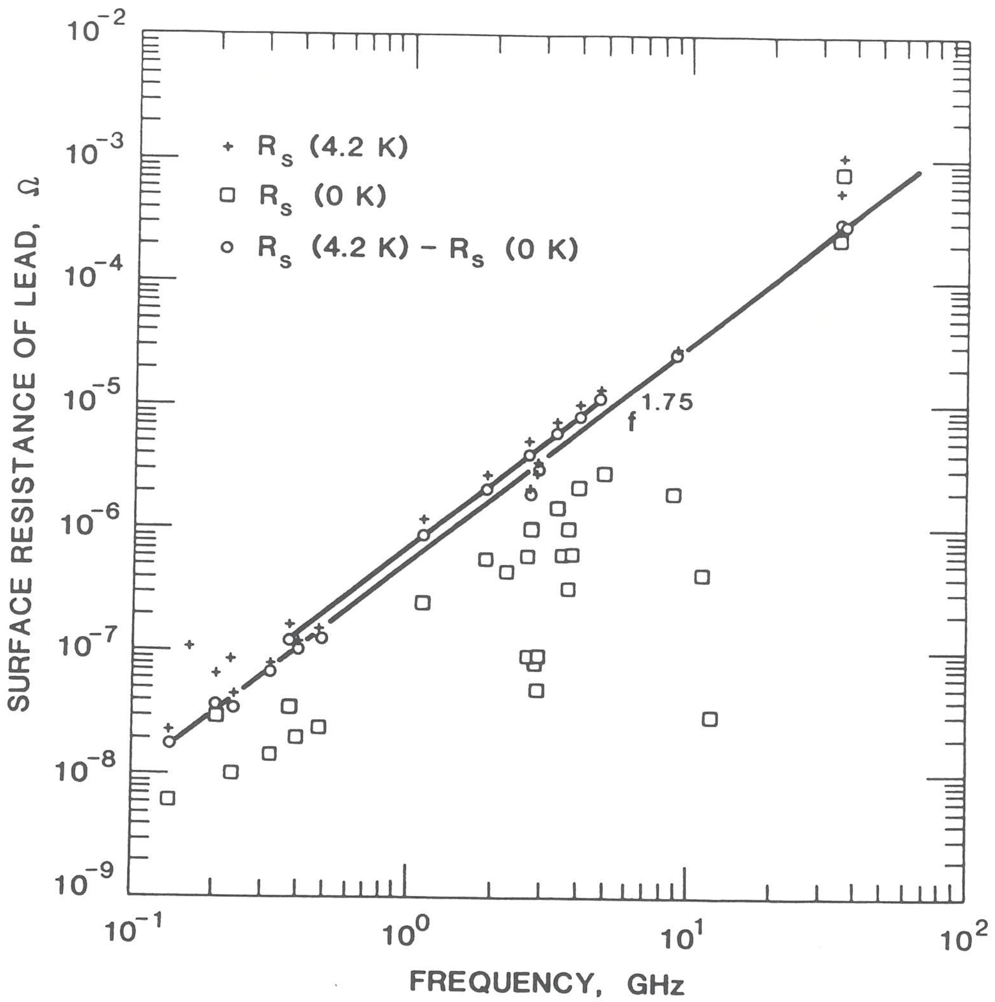


Fig. 11. *BCS and residual surface resistance versus frequency for Lead plated resonators (After, J.R. Delayen[25]).*

The great advantage of normal accelerating cavities is that a particle beam driven across a chain of them can be accelerated to very high energy by small RF fields, i.e. without need of high static voltages.

The advantage of superconducting accelerating cavities is twofold since it joins to the previous one the fact that RF electric fields of several MV/m can be sustained with only few watts of power losses.

For clearness purposes we like to remind the working principle of accelerating cavities. If we consider the pill-box cavity discussed above and mount two beam ports on the capacitance plates coaxially to the can, where the maximum electric field is present (fig. 12), a particle bunch crossing the cavity will be accelerated provided that its velocity is synchronized with the radio-frequency phase and period.

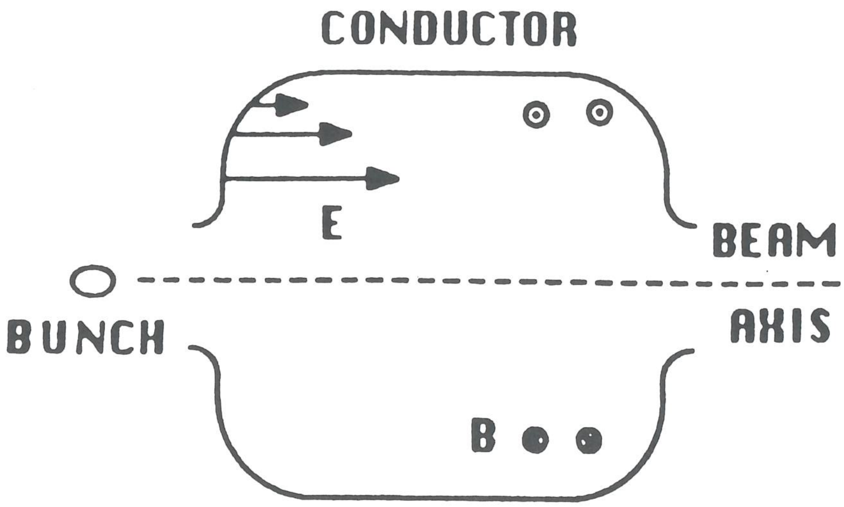


Fig. 12 *A single cell resonant cavity.*

Let us construct now a module of coupled cavities disposed in cascade as in fig. 13.

Of course there are different modes at which one can excite this system. The one represented in fig. 13 is the so called π-mode that is the favourite operation mode of accelerating structures.

This situation founds a mechanical analogue in a coupled pendula system[27)]. In fig. 14 are displayed the zero-mode and the π-mode among all those permitted to this system.

Coming back to accelerating cavities, if the velocity of the particle bunch is synchronized with frequency and phase of rf fields when entering into the resonator, a perfect feature can be realized: the bunch is accelerated when crossing the first cell; in the meanwhile in the second cell the field is decelerating, but it becomes accelerating one semi-cycle later just when the beam arrives. The same holds for the other cells and the beam gets accelerated in each cell.

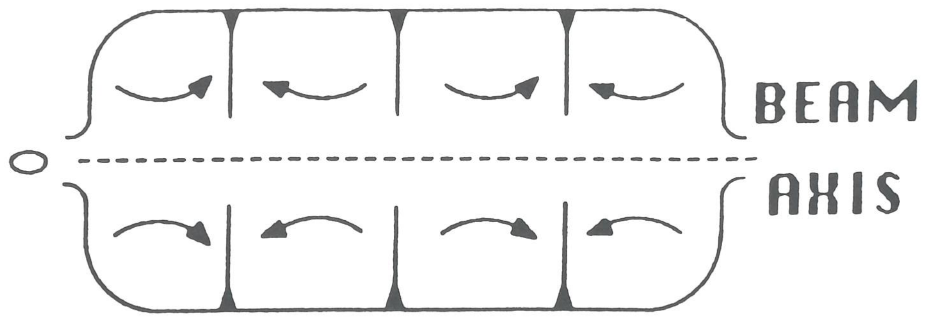


Fig.13 *A multi-cell cavity module excited in the π-mode. This mode is characterized by the fact that the accelerating fields in each pair of cavities are equal in magnitude and opposite in direction.*

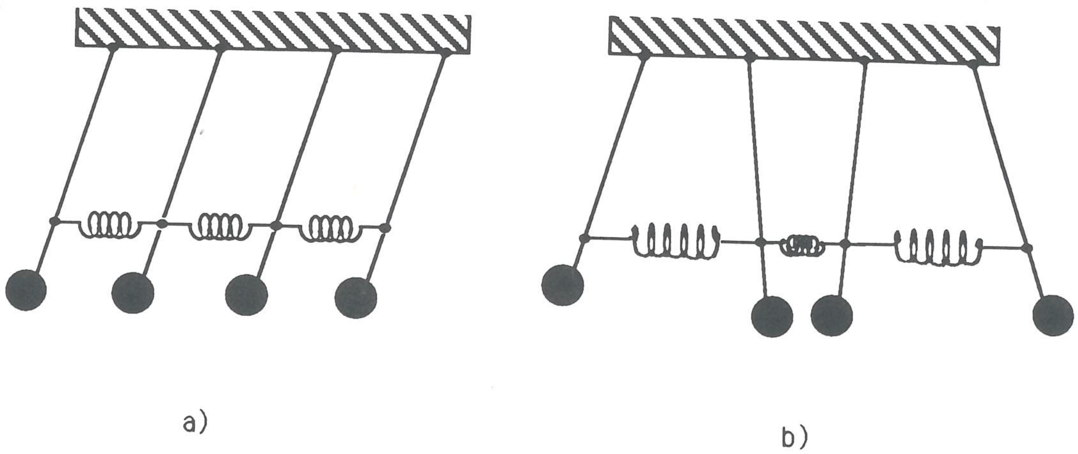
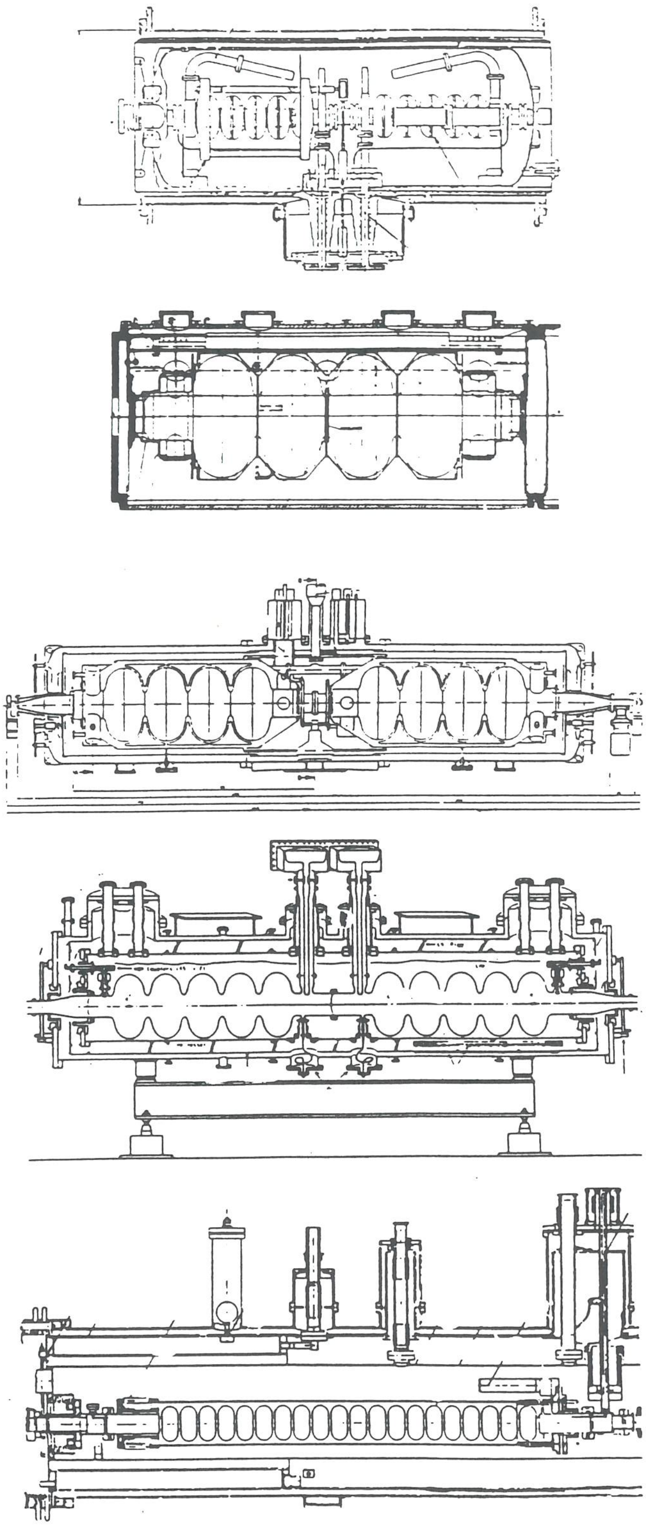


Fig. 14 *The coupled pendula analogue:*
a) The zero-mode: The phase difference is zero. The oscillation frequency is that of one independent pendulum.
b) The π-mode: The phase difference is π. The oscillation frequency is changed by the spring.



a

b

c

d

e

Fig. 15 *The superconducting electron cavity module with cryostat used at: a) CEBAF for the Superconducting Recirculating Linac, b)CERN for LEP, c) DESY for HERA, d) KEK for TRISTAN, e) Darmstadt for S-Dalinac*

When considering again resonator of fig. 12, we observe that its effective length **d** of the cavity must be adjusted to the particle velocity $v = \beta c$ and to the resonant frequency ω_0 and because the field accelerates only during one cycle, it holds

$$\omega_0 \, d \leq \pi \beta c \, .$$

That means that typical resonant frequencies for heavy ions cavities **($0.05 < \beta < 0.2$)**, range between 50 and 150 MHz since **d** has values generally between 5 and 10 cm. On the other side the typical resonant frequencies for electron cavities ($\beta \sim 1$) orbit around values between 350 and 3000 MHz.

The ratio β determines the shape of the cavity. Because of that different electron cavities used in the different high energy accelerators around the world (fig. 15) can differ for frequency or number of cells, but they have practically the same shape.

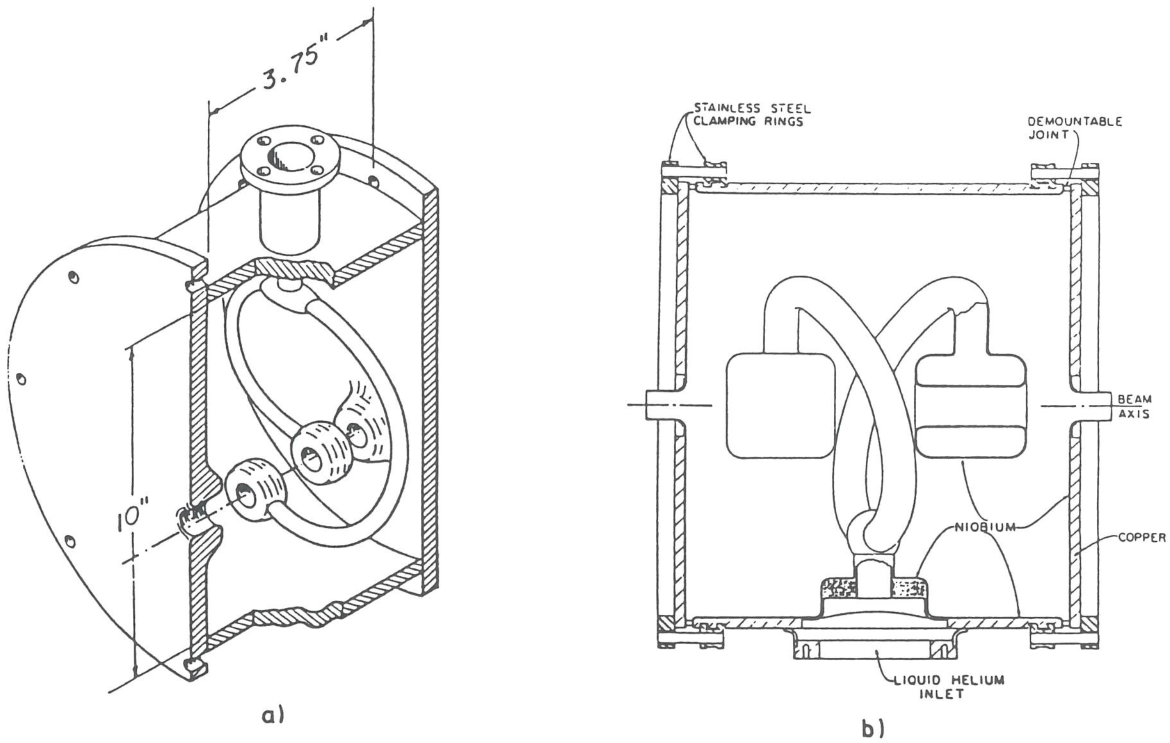


Fig. 16 *Split-ring resonators of a) Calthec/Stony Brook design*[28)] *and b) the one used in Argonne*[29)].

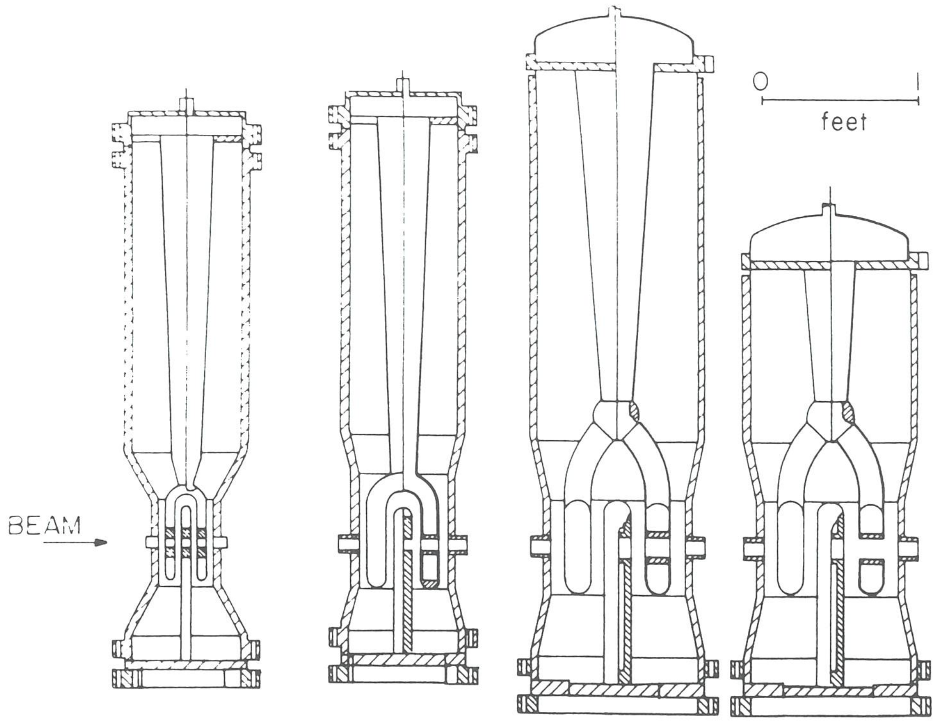


Fig. 17 *Four-gap interdigital superconducting resonators of different β and ω_0 for the Argonne injector linac*[30]*.*

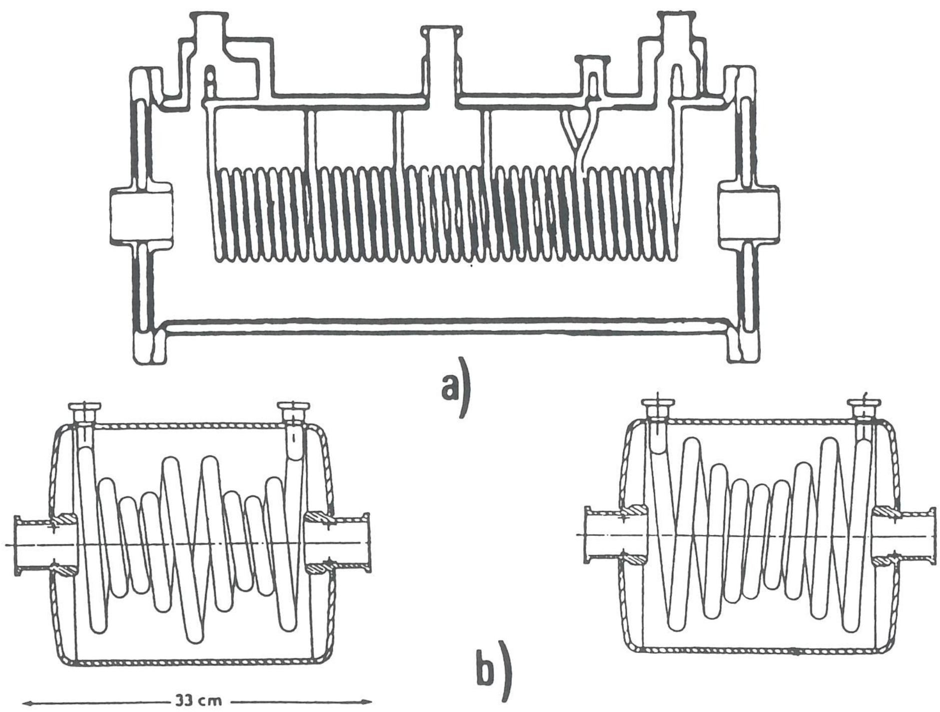


Fig. 18 *a) The superconducting helix resonator for the proton accelerator at Karlsruhe*[31]*. b) The helix resonators for the heavy ion accelerator for the CEN Saclay*[32]*.*

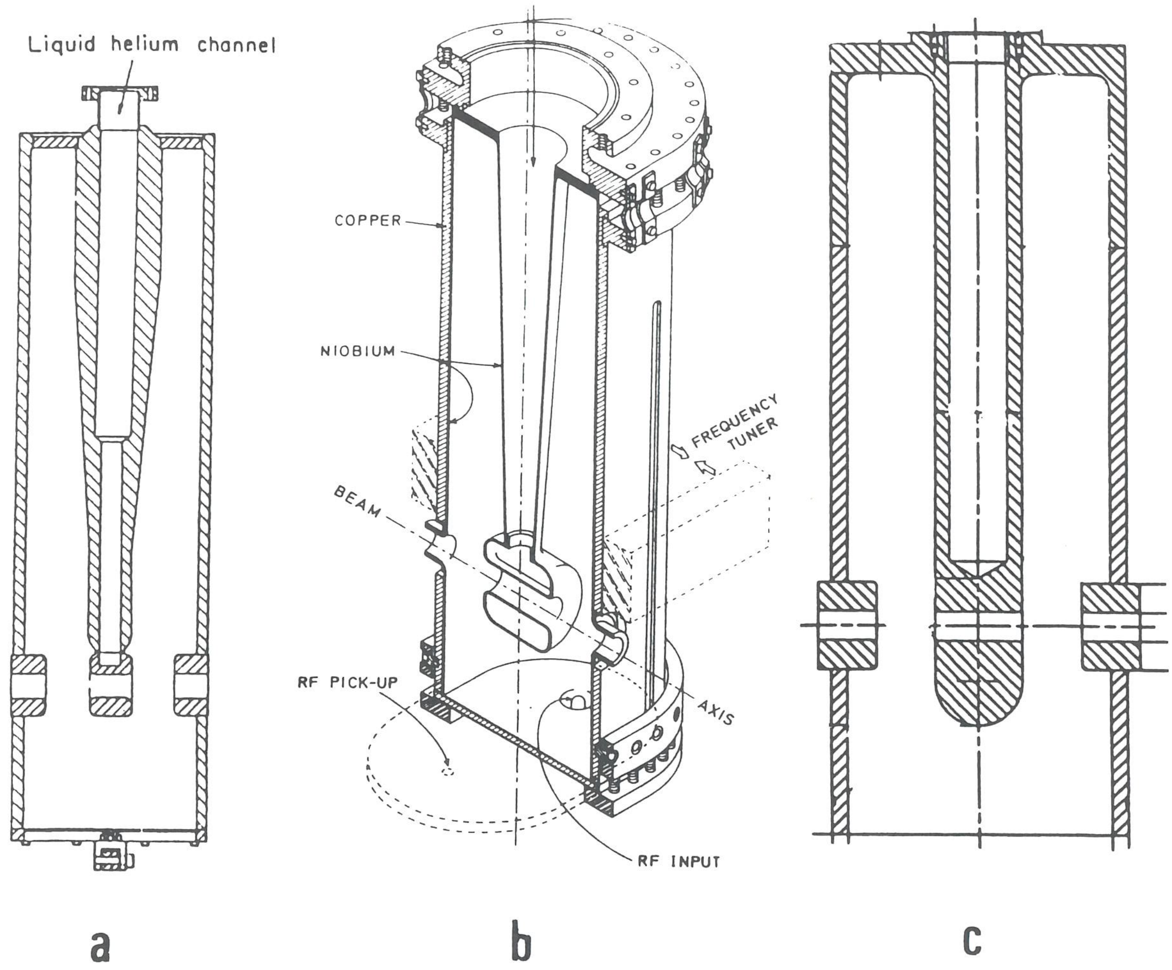


Fig.19 *Quarter Wave Resonators for*

a) the Washington University superconducting linac[33] *in Seattle.*

b) the JAERI booster in Japan[34]*.*

c) the ALPI superconducting heavy ion linac[35] *under construction in Legnaro.*

On the contrary several families of low-beta superconducting cavities exist and within each family cavities differ depending on β and ω_0.

Different types of split ring cavities are for example pictured in fig 16., while in fig. 17 the interdigital family is sketched.

Fig. 18 displays the helixes and in fig, 19 some quarter wave resonators are shown in fig. 19. In fig. 20 instead half wave resonators appear.

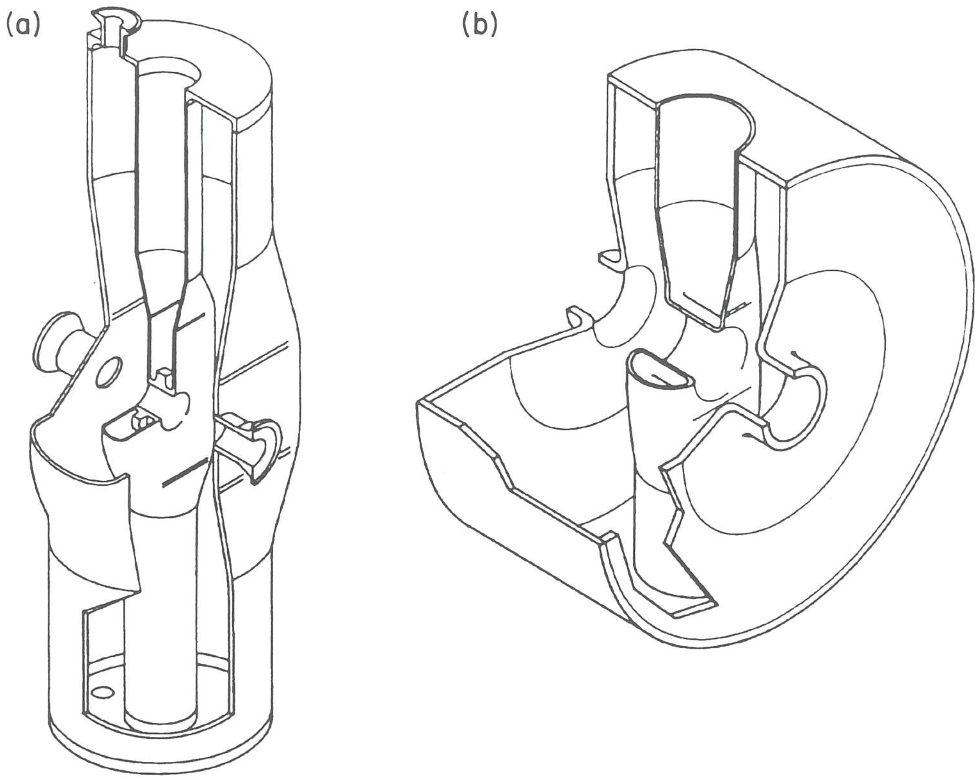


Fig.20 *Half wave resonators for the acceleration of light ions developed in Argonne National Laboratory*[30]. *The type a) has* $\omega_0 = 352$ *MHz and* $\beta = 0.12$; *the type b) has* $\omega_0 = 850$ *MHz and* $\beta = 0.28$.

5. CONCLUSIONS.

This paper wants to be just an introduction to the subject of superconducting resonant devices and it is far away from being exhaustive. There are many topics that are of real enjoyment, but here they were not treated. In particular it is worthwhile to mention two of them:

1) The application of the sputtering technique to the superconducting accelerating cavities together with the innovative superconducting materials that by means of this technique are considered of interest for new generation accelerating cavities.

2) The various applications of low-power resonators like filters, oscillators, detectors, amplifiers, antennas, transmission lines or superconducting resonators for material research.

Unfortunately giving them the same weight of the other topics treated above would make this article not readable. I can only say that, in my opinion they are a real mine of discoveries and new ideas.

REFERENCES:

The main purpose of this work was to maintain a pretty didactic character. This has been attempted also collecting in some parts of it several concepts from different reviews or different books.

Hence for example the construction of a pill-box starting from a RLC circuit comes from ref. 1 and the pendula analogue for a cavity from ref.27. I love them so much that it would have been a pity to skip them.

1) R.P. Feyman, "The Feyman lectures on Physics", vol.II, Addison Wesley Publ. Comp. p.23-11 (1965).

2) W.Weingarten, "Superconducting Cavities", CERN/AT-RF 91-9, 1 July 1991 presented at the CERN Accelerator School, Oxford, April 91.

3) M. Tinkham, "Introduction to superconductivity", Mc Grow Hill, (1975), p. 69.

4) D.C. Mattis, J. Bardeen, Phys. Rev. 111, 412 (1958).

5) S.B. Nam, Phys. Rev., 156, 487 (1967).

6) R.L. Kautz, J. Appl. Phys., 49, 309 (1978).

7) J. Halbritter, J. Appl. Phys., 42, 82, (1971).

8) J. Halbritter, J.Less. common metals 139, 133 (1988).

9) P. Kneisel, O. Stoltz, J. Halbritter, J. Appl. Phys., 45, 2296 (1974).

10) J. Halbritter, Phys. Lett. 49A, 397 (1974).

11) J. Halbritter, IEEE Trans. Mag., MAG-11, 427 (1972).

12) C. Passow, Phys. Rev. Lett., 28, 427 (1972).

13) K. Scharnberg, J. Appl. Phys. 48, 3462 (1977).

14) E. Kartheuser, S. Rodriguez, J. Appl. Phys., 47, 700 (1976) and J. Appl. Phys. 47, 3651 (1976).

15) M. Rabinowitz, Appl. Phys. Lett. 19, 73 (1971).

16) J.M. Pierce, J. Appl. Phys. 44, 1342 (1973).

17) J. Gilchrist, Proc. Roy. Soc., A 295, 399 (1966).

18) G. Arnolds- Meyer et al, CERN-EF/RF 86-1, (4 March 1986).

19) A.M. Portis, "Microwaves and Superconductivity: Process in the intergranular coupling", Springer Series in Solid State Sciences, ed. J. Bednorz and K.A. Muller, Springer verlag, Heidelberg (1989).

20) T.L. Hilton, A. Kapitulnik, M.G. Beasley, J.P. Carini, L. Drabeck, G. Gruner, Appl. Phys. Lett. 53, 1343 (1988).

21) C. Attanasio, L. Maritato, R. Vaglio, Phys Rev. B 43 (Feb. 1991).

22) Proceeding of the 5-th RF Superconductivity Workshop, Hamburg, (August 1991) to be published.

23) M. Peineger et al, 3-th RF Superconductivity Workshop, Argonne, (September 1987), p. 503.

24) M. Becks et al., 4-th RF Superconductivity Workshop, Tsukuba, (August 1989), p. 109.

25) J.R. Delayen, ibid. ref. 23, pag. 469.

26) H.A. Schwettman et al, Advan. in Cyogen. Eng. 10, Sect. M-U, 88 (1965).

27) H. Piel, "Superconducting Cavities" on Proc. of "Superconductivity in Particle Accelerators", CERN Accel. School, S. Turner ed., Hamburg, June 88/CERN-89-04 (10 March 1989) p. 149.

28) G.J. Dick et al. Nucl. Istr. and Meth. 138, 203 (1976).

29) K. W. Shepard et al, IEEE Trans. Nucl. Sci. NS-24, 1147 (1977).

30) K. W. Shepard, ibid. ref. 24, p.140.

31) A. J. Citron, et al, Proc. Int. Conf. High Energy Acc., 8th, CERN, Geneva (1971).

32) B. Cauvin, ibid. ref. 23, p. 379.

33) D.W. Storm et al, ibid. ref. 23, p. 367.

34) S. Takeuchi et al, ibid. ref. 24, p. 469.

35) G. Fortuna et al, ibid. ref. 23, p. 399.

36) Van Duzer, T., Turner, L., "Principles of Superconductive Devices and Circuits, North-Holland (1981).

37) H. Padamsee, Journal of Superconductivity, 1, 4, 377 (1988)