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TOWARDS AN ULTRACRYOGENIC BAR OBSERVATORY FOR GRAVITATIONAL WAVE BURSTS

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Abstract We propose a novel approach to the autonomous detection and reconstruction of a gravitational wave bursts by an intercontinental network of six resonant antennae. Our method consists first in determining the arrival times in at least three antennae and then in recombining the responses of all the six antennae on the same wavefront. The method gives amplitude, direction of propagation and polarization of the burst. In addition it provides a linear and a cubic veto that effectively reject as spurious non gravitational wave bursts those signals which are not traceless and transverse.

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1. Introduction

A generation of gravitational wave antennae, resonant ultracryogenic [1,2,3] and interferometric [4,5], is expected to be in operation in the second half of the 90's, with sensitivities which should be sufficient to detect the most energetic events predicted by current astrophysics [6,7]. In particular resonant antennae should be more apt to the detections of gravitational wave bursts, as those predicted with numerical gravitation methods to occur in type II supernova collapses, when either a vibrating black hole [8] or a neutron star [9] is formed.

Taking as reference these predictions, which consistently give a total energy emission in gravitational waves of the order of 10^{-3} $M_{\odot}$ per event and taking into consideration the burst sensitivity, defined as the minimum amplitude of "standard pulse" detectable at unity signal to noise ratio, $h_{min} \approx 3 \times 10^{-20}$, as calculated [10] with an electromechanical model for the AURIGA ultracryogenic antenna, we estimate [11] that some 13 galaxies with masses larger than $\approx 1/10$ of the Galaxy, are in range, ≈ 5 Mpc, to produce detectable supernova signals. Given the predicted rates of type II supernova explosions in galaxies [12], the expected rates of detectable signals should significantly increase, with respect to the predicted rates of Galactic supernova events, to approach the not unreasonable value of one per year.

The strategy planned to assign unequivocally to gravitational waves the signal detected by the few antennae in operation consists in making coincidences, within the optimal post–detection bandwidth [13]. Of course this strategy will be the only one to be implemented as soon as two or three ultracryogenic antennae will be in contemporary operation. However such a strategy has limits, which may weaken considerably the confidence in assigning events. First, even in the ideal case in which the statistics of noise is perfectly known, it is easy to see that three antennae, with thresholds set at $\approx 3h_{min}$, will go in coincidence, just because of thermal noise, at least once per year, a figure comparable with the expected event rate. Second, the optimal postdetection bandwidth are calculated to be at most $10 \div 50$ Hz; even if the antennae are far apart on Earth, still it will be difficult to look for light–time delays among them. Third, other signals as the light emission in the visible and in X and γ rays, the neutrino flash, etc. may be either undetectable or show up on so different time scales, that it may be difficult to confidently correlate to the few $msec$ duration of the gravitational wave burst. It should also be noticed that, taking as reference the neutrino emission in occasion of the $SN1987A$, the largest supernova neutrino detectors under construction will not be able to see extragalactic neutrinos. This is particularly disturbing because the neutrino flash from a type II supernova develops in some 10 sec, a time scale much more amenable to a coincidence with a gravitational wave burst than, for instance, the rise of the visible luminosity.

Thus should one be in the condition to receive candidate signals, candidate in the sense that they correspond to a coincidence of say three antennae, but which are not in correlation of any other astrophysical observation, it seems to us that it might be difficult to put up the case for a confident detection of gravitational waves.

These considerations have motivated us to explore the possibilities of a strategy of detection such that the properties of the Riemann tensor of the gravitational perturbation, which are distinctive to characterize a gravitational wave, would be recognized in a suitable combination of the response of a (minimal) number af resonant antennae, suitably located on the surface of the Earth. We have explicitly developed these ideas to propose such a network of ultracryogenic resonant antennae to behave as an intercontinental observatory. The observatory would be able to assign autonomously and unequivocally a multiple coincidence event within the network as the detection of a gravitational wave burst. Moreover it would give amplitude, direction of propagation and polarization of the gravitational wave burst, would measure the velocity of propagation of the perturbation and would give internal vetos uniquely obeyed by gravitational waves.

2. Distinctive properties of the action of a gravitational wave Riemann tensor on a network of resonant antennae

The Riemann tensor of a plane gravitational wave in the linear approximation can be easily shown to be, in the Transverse Traceless (TT) system X, Y, Z

$$R^0{}_{i0j} \equiv -\frac{1}{2c^2}\ {}^{TT}\ddot{h}_{ij}, \tag{2.1}$$

where $h_{\mu\nu}$ is the metric perturbation in the weak field approximation $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Choosing the Z-axis in the direction $\vec{k}$ of propagation of the wave and the X-axis along the major axis of the polarization ellipse (as in ref. [14]) we have

$$\begin{aligned} {}^{TT}h_{ij} &= h_+(t)\big(\vec{e}_X \otimes \vec{e}_X - \vec{e}_Y \otimes \vec{e}_Y\big)_{ij} + 2h_\times(t)\big(\vec{e}_X \otimes \vec{e}_Y\big)_{ij} \\ &\equiv h_+(t)(\mathbf{e}_+)_{ij} + h_\times(t)(\mathbf{e}_\times)_{ij}, \end{aligned} \tag{2.2}$$

where $\vec{e}_X$ and $\vec{e}_Y$ are the unit vectors in the direction of X and Y axes while $\mathbf{e}_+$ and $\mathbf{e}_\times$ are the polarization tensors with the properties of being transverse ($\mathbf{e}_{ij}k^i = 0$) and traceless ($\mathbf{e}_{ii} = 0$). As it is well known the Riemann tensor is gauge invariant (as its background value is zero) and therefore its properties are the same in any other reference system in particular in the reference frame of the antenna (i.e. the locally inertial rest frame of its center of mass). In this frame the equation of the fundamental mode of the bar is given by (see e.g. ref. [1])

$$\ddot{\xi} + \beta\dot{\xi} + \omega_0^2\xi = -\frac{4}{\pi^2}c^2 L R^0{}_{i0j} n^i n^j = \frac{2L}{\pi^2}\, {}^{TT}\ddot{h}_{ij}\, n^i n^j, \tag{2.3}$$

where L is the length of the bar, $\vec{n}$ is the unit vector parallel to the axis of the bar and ξ is the displacement induced by the wave in the $\vec{n}$ direction. Given an orthogonal reference frame with z' axis in the direction of the axis of the antenna, the contraction between the tensors $n^i n^j$ and $\mathbf{e}_{ij}$ gives the usual antenna pattern $\sin^2\theta\cos(2\psi)$ for $+$ polarization and $\sin^2\theta\sin(2\psi)$ for $\times$ polarization where θ, ψ are two of the three Euler angles between the two systems of coordinates: θ is the angle between Z and z', ψ between X and the line of the nodes [15] (the third angle ϕ, between the line of the nodes and x', does not appear in the antenna pattern because of the cylindrical symmetry of the antenna itself). From a simple calculation it follows that for unpolarized radiation a gravitational antenna "covers" (in the sense that the energy coming from any source in that region is at least half of the maximal one) about 1/2 of the celestial sphere. It would seem that this dependence on the angles were particularly unfavourable for the search of gravitational waves but one should notice that, as it reflects the tensor nature of the gravitational force (spin two graviton), it can be used to discriminate between gravitational waves and other physical disturbances, as we shall see below.

Let us consider several antennae that operate in coincidence. For the full reconstruction of a gravitational signal it turns out particularly useful the reference frame x, y, z which we shall call geocentric. It is defined with its origin in the center of the earth, z axis pointing to the north pole, x axis in the direction of the γ point of Aries and y axis orthogonal to them and forming a left-handed system (see Fig. 1). To this system we shall refer both the source and the detectors of the network. In fact one can easily connect the position of a source in these coordinates with the usual equatorial coordinates right ascension R.A. and declination δ by means of

$$\begin{cases} \Phi = R.A. - \pi/2 \\ \Theta = \delta + \pi/2, \end{cases} \tag{2.4}$$

where Θ, Φ are two of the three Euler angles between the TT and the geocentric systems (the third angle Ψ, between the node line and X axis is the polarization angle); Θ is the angle between Z and z, Φ is the angle between the node line and x. We need also the orthogonal transformation matrix O_{ij} between the two systems given by (see e.g. ref. [16])

$$\begin{pmatrix} \cos\Phi\cos\Psi - \sin\Phi\sin\Psi\cos\Theta & -\sin\Phi\cos\Psi\cos\Theta - \cos\Phi\sin\Psi & \sin\Phi\sin\Theta \\ \cos\Phi\sin\Psi\cos\Theta + \sin\Phi\cos\Psi & \cos\Phi\cos\Psi\cos\Theta - \sin\Phi\sin\Psi & -\cos\Phi\sin\Theta \\ \sin\Psi\sin\Theta & \cos\Psi\sin\Theta & \cos\Theta \end{pmatrix}. \tag{2.5}$$

In the geocentric system the polarization tensors are therefore expressed by $e'_{(+,\times)ij} \equiv O_{ik} e_{(+,\times)kl} O^{-1}_{lj}$.

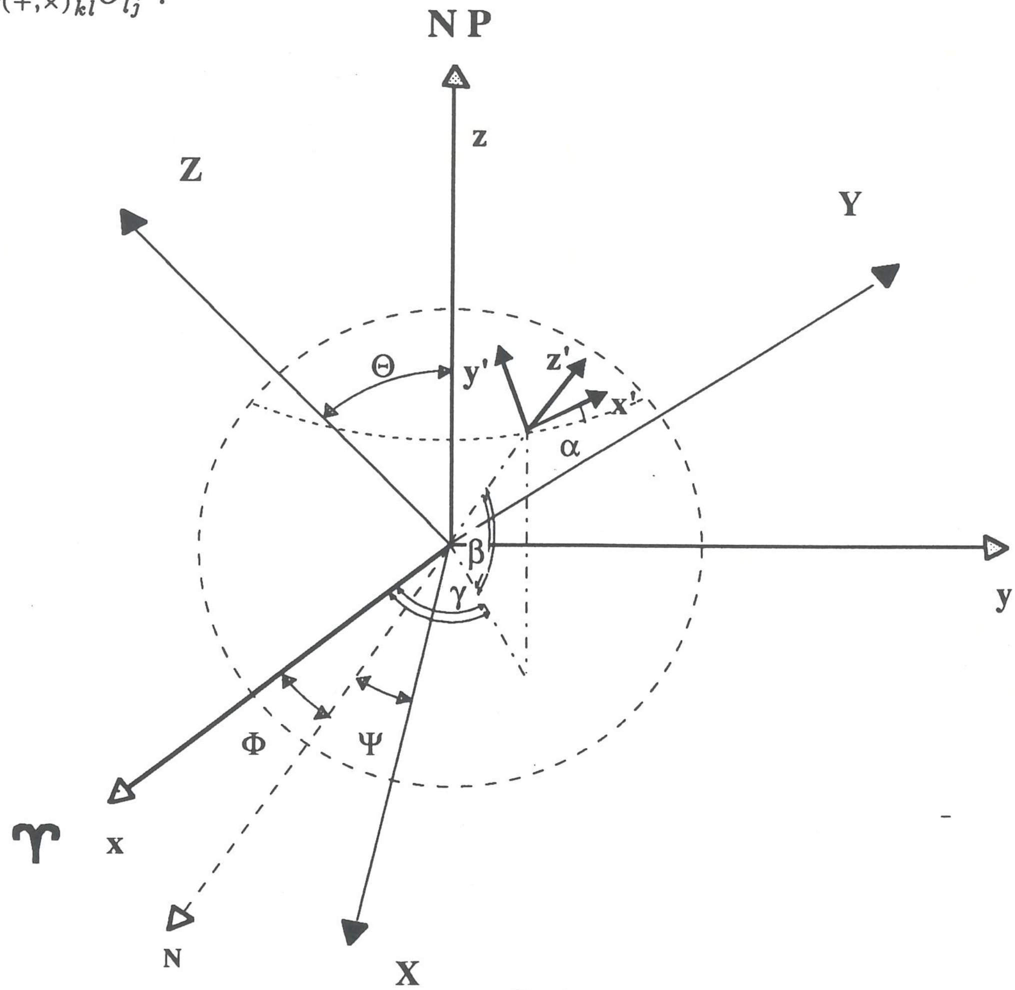

Fig. 1

The definition of the antenna axes (x', y', z'), the gravitational wave axes (X, Y, Z) and the geocentric axes (x, y, z). β and γ are the latidute and longitude of the antenna, respectively, and α is the angle between the bar axis and the local Est–West direction. Θ and Φ give the direction of the wave with respect to geocentric system while Ψ determines the polarization angle.

If we call α the angle which the detector makes with east-west direction, β its latitude and $\gamma = \omega_\oplus t$ (where $\omega_\oplus$ is the angular velocity of the earth and t is the Universal Time) the Euler angles between the detector system x', y', z' (where x' is parallel to $\vec{n}$, z' is the vertical and y' is such as to form a left-handed orthogonal system) and the geocentric system are given by (see Fig. 1)

$$\begin{aligned} \theta &= \pi/2 - \beta \\ \phi &= \gamma - 3\pi/2 \\ \psi &= \alpha - \pi/2. \end{aligned} \tag{2.6}$$

In this system the antenna pattern depends on the detector angles (α, β, γ), which are known, and on the source angles (Θ, Φ, Ψ), which are unknown and to be determined by our method together with the amplitude of the wave. At first glance it would appear that the unknowns in the problem are five: the two amplitudes $h_+(t)$ and $h_\times(t)$ and the three Euler angles Θ, Φ and Ψ but actually $h_+(t)$, $h_\times(t)$ and Ψ are not independent as we can choose X and Y axes such as it holds, at any time, the equation $\tan(2\Psi) = h_+/h_\times$. As the unknowns of the problem are 4 it would appear that 4 antennae would suffice to solve it completely. This is true when one already assumes that the signal is transverse and traceless i.e. it is indeed a gravitational wave. However in the experimental situations in which the signal may be as well a disturbance of unknown origin these two conditions must be tested in order to have confidence that a gravitational wave is actually being seen. This entails that the minimum number of antennae must be six. In this case it turns out also that the problem can be dealt with algebraically without having recourse to transcendental equations associated with the antenna pattern.

We now show how it is possible to solve the inverse problem for gravitational plane waves from the responses of 6 detectors. We consider first the case of six noiseless antennae located in the same place and then, in §4, we relax this condition and we consider six noisy antennae arranged in an intercontinental network.

The response of each detector (labelled with $\alpha = 1 \ldots 6$) is given by

$$R^\alpha(t) = (n^i n^j)^\alpha \int_0^\infty H(t-\tau) \;\; {}^{TT}\ddot{h}_{ij}(\tau) d\tau; \tag{2.7}$$

where H(t) is the transfer function of the antennae that in the simple harmonic oscillator approximation reduces to Eq. (3.1) below; of course the output of each antenna is obtained adding to the R^α the noise η^α which will be fully described in the next section.

We consider now linearly polarized radiation (as it is expected to come from supernova explosions evolving in black–holes [8]); what follows, however, still holds for other states of polarization or even for unpolarized radiation. In order to simplify further our notation we write

$$^{TT}h_{ij} = h_0 h(t) O_{ik} e_{+kl} O^{-1}_{lj} \equiv h_0 h(t) W_{ij}, \tag{2.8}$$

which, substituted in (2.8), gives

$$R^{\alpha} = A\xi(t) W_{ij} (n^i \, n^j)^{\alpha} \qquad \alpha = 1 \ldots 6 \tag{2.9}$$

where $\xi(t) = \int_0^{\infty} H(t-\tau)\ddot{h}(\tau)$ and $A = 2h_0 L/\pi^2$.

If the antenna axes $\vec{n}^{\alpha}$ are not parallel and no three of them lie with their axis on the same plane, we can write the tensor W_{ij} as a linear combination of the responses R^{α}

$$A\xi(t) W_{ij} = \sum_{\alpha=1}^{6} B^{\alpha}_{ij} R^{\alpha}(t), \tag{2.10}$$

where B^{α}_{ij} are six 3×3 matrices which depend only on the set of n^{α} and which can be determined by solving the linear system $\sum_{ij} B^{\alpha}_{ij} \, (n^i n^j)^{\beta} = \delta^{\alpha\beta}$. There exist three quantities which are invariant under the rotation group i.e. $Tr(W)$, $Tr(W^2)$ and $Tr(W^3)$. These quantities, because of the distinctive properties of the gravitational wave Riemann tensor, must be respectively equal to 0, 2 and 0. Any combination of such invariants is itself an invariant (for instance the determinant of W is equal to $Tr(W^3)/3 - Tr(W^2)Tr(W)/2 + Tr(W)^3/6$). So we have the following invariants in terms of the R^{α}

Linear

$$\mathcal{T} = A\xi(t) Tr(W) = \sum_{\alpha=1}^{6} Q_{\alpha} R^{\alpha}(t), \tag{2.11}$$

where $Q_{\alpha} = Tr(B^{\alpha})$ are the solutions of the algebraic system $\sum_{\alpha} Q_{\alpha} (n^i n^j)^{\alpha} = \delta^{ij}$;

Quadratic

$$\mathcal{H} = \frac{1}{2} A^2 \xi^2(t) Tr(W^2) = \frac{1}{2} \sum_{\alpha,\beta=1}^{6} Q_{\alpha\beta} R^{\alpha}(t) \, R^{\beta}(t), \tag{2.12}$$

where $Q_{\alpha\beta} = Tr(B^{\alpha} B^{\beta})$ are the solutions of the algebraic system $\sum_{\alpha\beta} Q_{\alpha\beta}$ $(n^i n^j)^{\alpha} \, (n^k n^l)^{\beta} = \delta^{ik} \, \delta^{jl}$;

Cubic

$$\mathcal{D} = A^3 \xi^3(t) Tr(W^3) = \sum_{\alpha,\beta,\gamma=1}^{6} Q_{\alpha\beta\gamma} R^{\alpha}(t) \, R^{\beta}(t) \, R^{\gamma}(t), \tag{2.13}$$

where $Q_{\alpha\beta\gamma} = Tr(B^{\alpha} B^{\beta} B^{\gamma})$ are the solutions of the algebraic system $\sum_{\alpha\beta\gamma} Q_{\alpha\beta\gamma}$ $(n^i n^j)^{\alpha} \, (n^k n^l)^{\beta} (n^r n^s)^{\gamma} = \delta^{is} \, \delta^{jk} \, \delta^{lr}$.

The linear invariant $\mathcal{T}$ can be used as a veto because it must be zero for a true gravitational wave. The mean of the quadratic invariant $\mathcal{H}$ is proportional to the energy released by the wave to the network and so its square root will give a measure of the amplitude of the wave. The cubic invariant $\mathcal{D}$ is a measure of the transversality of the wave: in fact $\mathcal{D} = 0$ is equivalent to the existence of a zero eigenvalue of W.

Obviously the condition that the antennae are located in the same place is incompatible with the requirement that no two of them be parallel and no three of them lie with their axis on the same plane as the antenna axes must always lie (for technical reasons) in the local horizontal plane. Our network should therefore be scattered over the Earth surface. This however entails that one must know the arrival times of the signal on the antennae in order to recombine the responses in the wavefront and therefore apply correctly the analysis of invariants. Moreover in order to have a realistic treatment of the behavior of the network one has to take into account the noise. This poses the problem of the determination of the arrival time which we solve in the next section.

3. Estimate of the arrival time of a gravitational signal

The ability to measure the arrival time of a pulse signal on a resonant antenna depends on the signal to noise ratio [17]. In order to better elaborate on this point we will consider a simplified model of the antenna in close analogy with the point of view suggested by Giffard [18]. In this model the antenna is considered as a simple harmonic oscillator with resonant angular frequency ω_0 and decay time τ, excited by a short pulse of force $f(t)$. The position $\xi(t)$ of the oscillator mass M is read by a suitable position transducer. The oscillator is driven in random motion both by the brownian force $f_B(t)$ and by the back–action force noise $f_{ba}(t)$ of the position transducer, this one contributing also an additive position noise $\xi_n(t)$. The brownian force has a white noise spectrum $S_B = 2k_BT\beta$, with β the coefficient of friction of the oscillator. Both $f_{ba}(t)$ and the added noise $\xi_n(t)$ are assumed to have white spectra with values S_{ba} and S_ξ respectively.

The model can be taken as a realistic approximation for a single mode of oscillation of the antenna-transducer-electric port system, as far as the mode is well isolated from the remaining ones. The model is expected to become somewhat inadequate when different modes come so close in frequency that they cannot be considered anymore as isolated. However we do not expect the basic conclusions of the following discussion to be very much affected by this complication. Within the above model the antenna acts as a linear device with $f(t)$ as the input and $\xi(t)$ as the output. Its impulse response is:

$$H(t) = \frac{1}{M\omega_1} \exp\left(-\frac{t}{\tau}\right) \sin(\omega_1 t) \tag{3.1}$$

with $\omega_1^2 = \omega_0^2 - 1/(4\tau^2)$, and its frequency response is

$$H(\omega) = \frac{1}{M} \frac{1}{\omega_0^2 - \omega^2 + i\omega/\tau} \tag{3.2}$$

The antenna output $\xi(t)$ is given by

$$\xi(t) = \eta(t) + \int_0^{\infty} H(t')f(t-t')dt' \tag{3.3}$$

where $\eta(t)$ is the total output noise, a stationary stochastic process assumed to be gaussian and zero mean. Its power spectrum, according to the model above, is

$$S(\omega) = (S_B + S_{ba})|H(\omega)|^2 + S_\xi = S_f|H(\omega)|^2 + S_\xi, \tag{3.4}$$

where we have introduced the total force power spectrum $S_f = S_B + S_{ba}$.

The estimation of the arrival time of a signal in the presence of noise is a well established problem in signal analysis [17]. If the shape of the signal is known and only the amplitude and the arrival time have to be estimated, the standard Wiener filtering theory can be applied. In the framework of this theory the force signal $f(t) = I_o g(t - t_a)$ contains an unknown amplitude I_o and an unknown time of arrival t_a. To estimate both I_o and t_a, one first selects a value of t_a, let us call it t_0, and builds, for that choice of t_a, the linear unbiased estimator of I_o of minimum variance:

$$\widehat{I}(t_0) = \int_{-\infty}^{+\infty} W(t_0 - t)\xi(t)dt \tag{3.5}$$

where the weight function $W(t)$ has a Fourier transform

$$W(\omega) = \sigma^2 \frac{g^*(\omega)H^*(\omega)}{S(\omega)} \tag{3.6}$$

the variance σ^2 being

$$\sigma^2 = \left\{ \int_{-\infty}^{+\infty} \frac{|g(\omega)|^2|H(\omega)|^2}{S(\omega)} \, d\omega \right\}^{-1} \tag{3.7}$$

The value $\widehat{t}_a$ of t_0 for which $|\widehat{I}(t_0)|$ reaches its maximum is then taken as an estimate of t_a.

In order to evaluate the uncertainty of the estimation of t_0, let us consider first that, in the absence of any signal ($I_o = 0$), $\widehat{I}(t_0)$ reduces to a zero mean stochastic process $\widehat{I}_n(t_0)$ with t_0 playing the rôle of the time. The power spectrum of this process is

$$S_{\widehat{I}}(\omega) = \sigma^4 \frac{|g(\omega)H(\omega)|^2}{S(\omega)} = \sigma^4 S_\xi^2 M^2 \frac{|g(\omega)|^2}{(\omega_\star^2 - \omega^2)^2 + \omega^2/\tau_\star^2} \tag{3.8}$$

where $\omega_\star^4 = \omega_1^4 + S_f/(S_\xi M^2)$ and $\tau_\star^{-2} = \tau^{-2} + 2(\omega_\star^2 - \omega_1^2)$. For real antennas $\omega_\star \approx \omega_1$ and $\tau_\star \approx M/\beta_n$, with $\beta_n = \sqrt{S_f/S_\xi \omega_1^2}$ a noise "impedance" with physical dimensions of a friction coefficient. $S_{\widehat{I}}(\omega)$ is the power spectrum of an harmonic oscillator, of frequency $\omega_\star$ and damping time $\tau_\star$, driven by a random force with spectrum $|g(\omega)|^2/(\sigma^4 S_\xi)$. If $|g(\omega)|^2$ varies slowly in the range range $\omega \approx \omega_\star \pm 1/\tau_\star$ then $|g(\omega)|^2$ can be substituted by $|g(\omega_\star)|^2$ in eq. (3.8) and the fluctuations of $\widehat{I}(t_0)$ consist in practice of oscillations at frequency $\omega_\star$ that keep their coherence on a typical time of order $2\tau_\star$. When the signal is present $\widehat{I}(t_0)$ will then result in a signal part

$$\widehat{I}_s(t_0) = \frac{I_0}{\sigma^2} \int_{-\infty}^{+\infty} \frac{|g(\omega)H(\omega)|^2}{S(\omega)} e^{i\omega(t-t_0)} d\omega \tag{3.9}$$

added to the noise part $\widehat{I}_n(t_0)$. If again $|g(\omega)|^2$ varies slowly in the range $\omega \approx \omega_\star \pm 1/\tau_\star$ then $\widehat{I}_s(t_0)$ is an oscillating function of $t_0 - t_a$, at frequency $2\omega_\star$ exponentially damped with time constant $2\tau_\star$. This function reaches its maximum for $t_0 = t_a$. This is obviously not true for $|\widehat{I}(t_0)| = |\widehat{I}_s(t_0) + \widehat{I}_n(t_0)|$ that will reach its maximum at time $\widehat{t}_a$. Separating the random part of $\widehat{t}_a$ we can write $\widehat{t}_a = t_a + t_r$. An analytic evaluation of the statistics of the zero mean random variable t_r in a few situations was obtained long ago [19]. To discuss the main conclusions that apply to our case let us first separate the "phase" part δt of t_r writing $t_r = \delta t + nT_\star/2$, with $T_\star = 2\pi/\omega_\star$, $\delta t \leq T_\star/4$ and n an integer. The first result is that the standard deviation of δt, $\sigma_{\delta t}$ is given by:

$$\sigma_{\delta t} = \frac{T_\star}{2\pi(SNR)}, \tag{3.10}$$

where the signal to noise ratio SNR is defined by $SNR = I_o/\sigma$. The second important remark is that for $SNR >> 1$ ($SNR \geq 2 \div 3$ with 10 % approximation) the standard deviation of n is given by

$$\sigma_n = \frac{Q_\star}{\pi(SNR)^2} \tag{3.11}$$

with $Q_\star = \omega_\star \tau_\star$, if again $|g(\omega)|^2$ can be considered a constant in the range $\omega \approx \omega_\star \pm 1/\tau_\star$. In this limit $I_s(t_0)$ shows a cusp for $t_0 = t_a$ that makes easier the arrival time detection.

An important case is that in which $g(t)$ is a wave packet with a center frequency close to ω_0 and a duration τ_p. This is probably the closest approximation to the expected gravitational wave signals. One finds that, if τ_p is shorter than $1/\omega_0$, then one recovers the result in Eq. (3.11). If instead τ_p is longer than that, and the signal has no cusps by itself, then one obtains for σ_n

$$\sigma_n = \frac{2\tau_p}{T_\star(SNR)} \tag{3.12}$$

where the duration τ_p is properly defined in Ref. [20] and is $\approx 2\Delta t$ for a gaussian $(\exp(-t^2/2\Delta t^2))$ envelope of the wave packet. Notice that as soon as $\sigma_n \leq 1$ the total uncertainty σ_t of the arrival time reduces to that in Eq. (3.10). For a 1 kHz antenna this gives $\sigma_t \approx 160/SNR$ μsec a time figure that easily allows time delay measurement for antennae at different places on the earth.

To better understand what is the meaning of σ_n we made a numerical simulation using $g(t) = \delta(t)$ for the signal. In Fig. 2 the statistic of t_0 is reported taking $t_a = 0$, $Q_\star = 30$ and $SNR = 10$. It can be easily seen that the data group in large majority (> 95 %) around $t_0 = 0$ while a few are found around $\pm T_\star/2$ and around $\pm T_\star$.

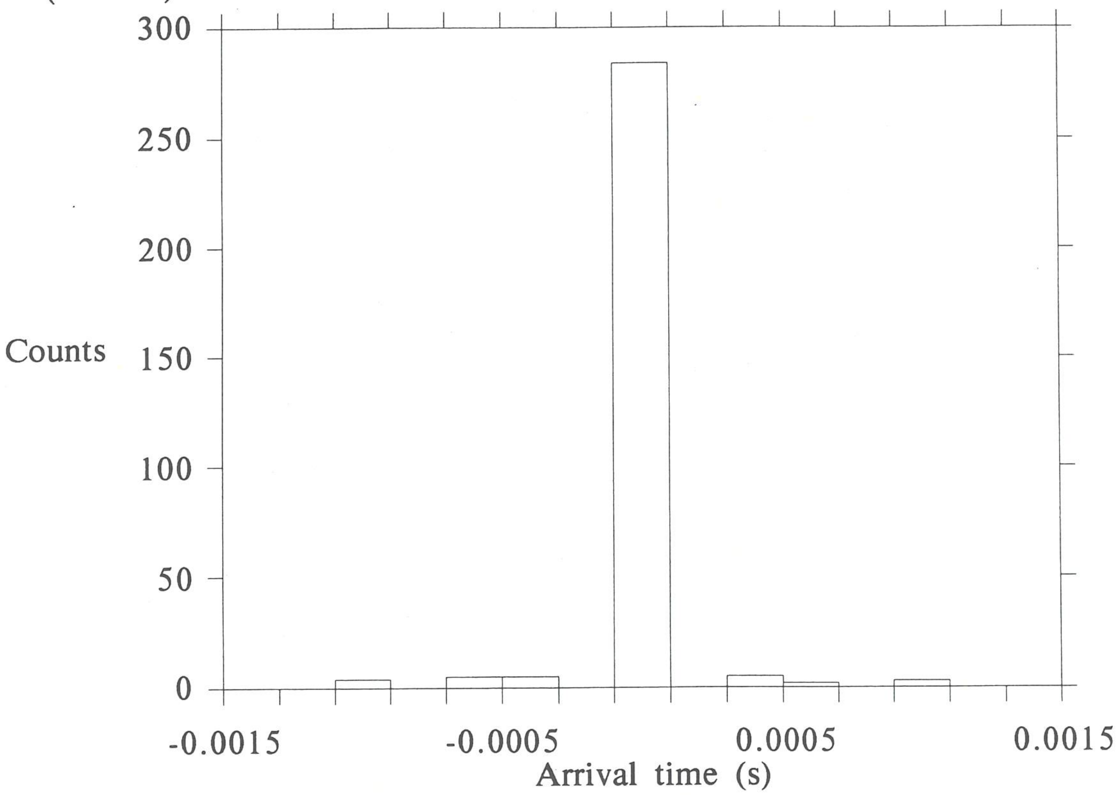

Fig. 2

Distribution of t_0 *(assuming* $t_a = 0$*) with* $SNR = 10$, $Q_\star = 30$ *and* $\omega_\star = 2\pi \times 10^3$ rad/sec.

The dependence of the value of σ_t, here defined as that value for which 70 % of the data give $t_r \leq \sigma_t$, is reported in Fig. 3 as a function of SNR for $Q_\star = 30$. This value of $Q_\star$ should be within the reach of the generation of detectors that are presently under construction [2]. It can be seen that as soon as SNR becomes larger than $SNR \approx 7$, σ_t switches from the value $\sigma_t = (T_\star/2)\sigma_n$, with σ_n given by Eq. (3.11), to $\sigma_t = \sigma_{\delta t}$ with $\sigma_{\delta t}$ given by Eq. (3.10). Numerical simulation [20] also allowed to clarify that Eq. (3.11), a much more favourable result than that in Eq. (3.12), still holds for gaussian packets up to $\Delta t = T_\star/2$.

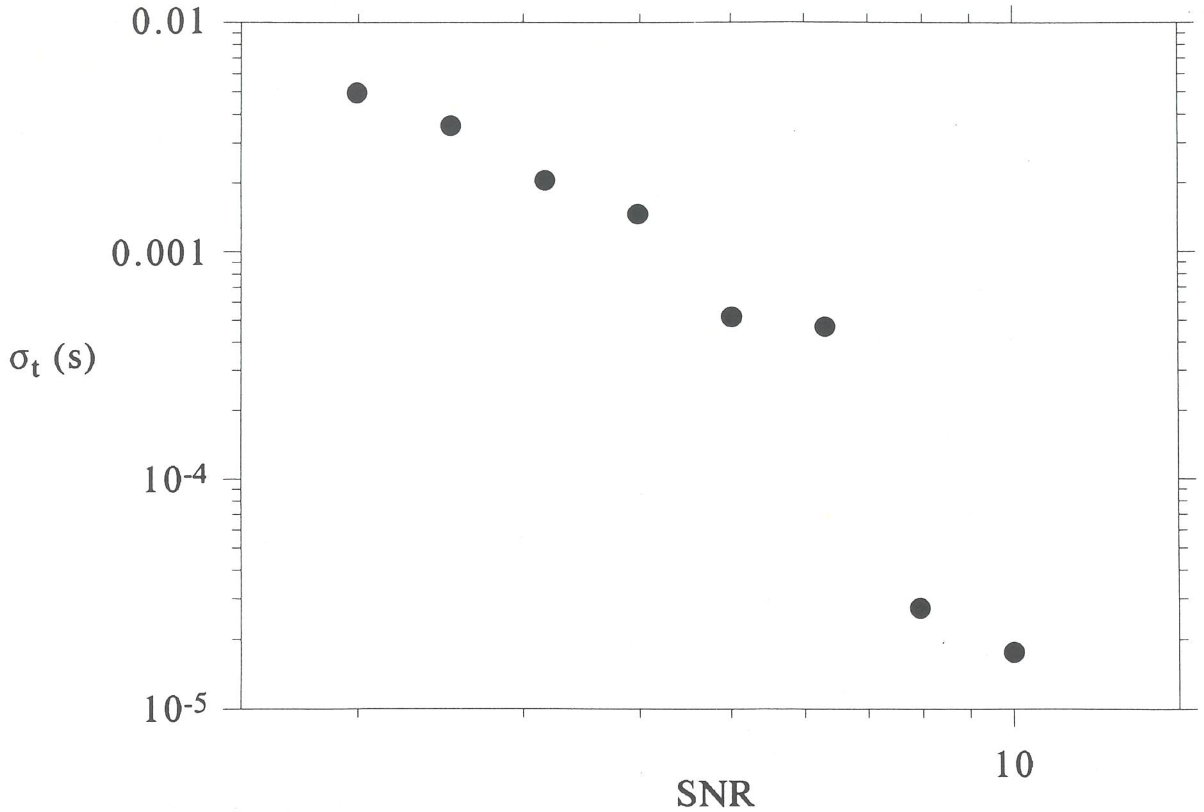

Fig. 3

Total uncertainity of arrival time σ_t as a function of SNR; $Q_\star =$ 30 and $\omega_\star = 2\pi \times 10^3$ rad/sec.

4. Detection of a gravitational wave by an intercontinental network of resonant antennae

In order to illustrate our detection strategy let us consider for simplicity a configuration in which the detectors are parallel to the lines joining the opposite faces of a dodecahedron. It is easy to see that this is possible on the Earth surface and let us call P^α the places where the antennae could be located. These directions cover almost isotropically the whole solid angle and make the calculations particularly simple. With respect to a suitable reference frame they can be written as $\vec{n}^1 = (0 \;\; 0 \;\; 1)$, $\vec{n}^2 = \left(0 \;\; -\frac{2}{\sqrt{5}} \;\; \frac{1}{\sqrt{5}}\right)$, $\vec{n}^3 = \left(\frac{\sqrt{2\sqrt{5}+10}}{2\sqrt{5}} \;\; \frac{1-\sqrt{5}}{2\sqrt{5}} \;\; \frac{1}{\sqrt{5}}\right)$, $\vec{n}^4 = \left(\frac{\sqrt{10-2\sqrt{5}}}{2\sqrt{5}} \;\; \frac{\sqrt{5}+1}{2\sqrt{5}} \;\; \frac{1}{\sqrt{5}}\right)$, $\vec{n}^5 = \left(-\frac{\sqrt{10-2\sqrt{5}}}{2\sqrt{5}} \;\; \frac{\sqrt{5}+1}{2\sqrt{5}} \;\; \frac{1}{\sqrt{5}}\right)$, $\vec{n}^6 = \left(-\frac{\sqrt{2\sqrt{5}+10}}{2\sqrt{5}} \;\; \frac{1-\sqrt{5}}{2\sqrt{5}} \;\; \frac{1}{\sqrt{5}}\right)$. By means of Eqs. (2.11), (2.12) and (2.13) we can easily verify the following identities

$$\mathcal{T} \equiv Tr(W) = \frac{1}{2}\sum_{\alpha=1}^{6} R^\alpha \tag{4.1}$$

$$\mathcal{H} \equiv Tr(W^2) + \frac{1}{2}Tr(W)^2 = \frac{5}{4}\sum_{\alpha=1}^{6} R^{\alpha\,2} \tag{4.2}$$

$$\mathcal{D} \equiv Tr(W^3) - \frac{3}{8}Tr(W^2)Tr(W) + \frac{3}{16}Tr(W)^3 =$$

$$= \frac{25}{32}\sum_{\alpha=1}^{6} R^{\alpha\,3} + \frac{15\sqrt{5}}{32}\sum_{\alpha\neq\beta\neq\gamma}^{6} \varepsilon_\alpha\varepsilon_\beta\varepsilon_\gamma R^\alpha R^\beta R^\gamma \tag{4.3}$$

where $\varepsilon_\alpha = 1$ for $\alpha = 1, 5, 6$ and $\varepsilon_\alpha = -1$ for $\alpha = 2, 3, 4$. If a gravitational wave of wavevector $\vec{k}$ impinges on this network we can determine, by means of Eq. (3.5) applied to each to the six detectors, the arrival times of signals for the three antennae which have the most favourable figure pattern for the given event (and so the best SNR). If almost three $SNRs$ are greater than 10 we can determine the arrival time with the precision given by (3.12) and then reconstruct the direction of propagation $\vec{k}$ and so estimate the arrival time of the signal on every detector of the network. The Wiener filter applied off–line to the data allows to extract the amplitude of the wave (multiplied by the corresponding figure pattern) at these times. Now all the invariants can be calculated. From our analysis one expects that $\mathcal{T}$ and $\mathcal{D}$ are not affected by the wave while $\mathcal{H}$ is greatly enhanced.

We have started to perform numerical simulations of the method. Gravitational wave bursts, in the form of "standard pulses", are sent with random directions and polarizations to the network of six antennae. The simulation provides the antennae with their thermal noise, as calculated previously for the AURIGA antenna [2]. The responses are analyzed as outlined above. Preliminary results confirm the expected performances. A full Monte Carlo calculation is in progress to evaluate directly the level of confidence with which all this can be done as a function of the SNR of the incoming signal.

5. Conclusions

We have proposed a method which in principle solves optimally the problem of the autonomous detection of gravitational wave bursts. The observatory, as it is evident, would have equal sensitivity for all the celestial sphere and for any polarization state. The price to be paid is to deal with signals of amplitude $h \gtrsim 10 h_{min}$.

The results we give here can be easily adapted to a network of interferometric antennae, provided their location on Earth in properly chosen. Our analysis differ from the solution of the inverse problem worked out in ref. [21], where a somewhat complementary point of view is taken: assuming the properties and the velocity of propagation of gravitational waves, the minimum number of antennae is used to reconstruct the gravitational wave burst. It should be noticed that also in this case it is requested to deal with signals of amplitude $h \gtrsim 10 h_{min}$.

From a practical point of view it may look not unreasonable, at least in cost, to scatter over the Earth surface some 10 ultracryogenic antennae to set up the observatory, allowing for some redundancies. Of course the primary problem to be solved would be to evolve the present generation of antennae to a generation of more rugged, simple to run, highly reliable ultracryogenic antennae.

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