

ON THE CALCULATION OF CLUSTER RADIOACTIVE DECAY PROBABILITY

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Abstract

The cluster radioactive decay theories are shortly discussed, describing the features of α -decay like theoretical approach, as being appropriate to take into account the nuclear structure of initial, final and emitted nuclei. A suitable method for computing the cluster overlap integral in the Woods-Saxon shell model is presented, as a first step to calculate the cluster emission probabilities.

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1 Introduction

The theoretical investigation of the so-called "exotic" or "new" radioactivity started in 1980 with the seminal article of Sandulescu, Poenaru and Greiner, ref. [1] and was boosted four years later by the experimental discovery of ^{14}C emission from ^{223}Ra , made by Rose and Jones [2].

Initially, the search for cluster radioactivity was limited to trans-Radium nuclei and to a few emitted clusters, like the isotopes of C, Ne, Mg or Si. Several authors indicated also other possible cluster radioactivity in regions with $A \leq 200$ and around $A \approx 110$ (refs. [3] and [4]). More recently, the discovery of ^{20}O radioactivity by Bonetti et al. ref. [5] and, probably, of ^{23}F , ref. [6], extended the number of different emitted clusters up to ten. This trend justifies the interest presented by the theoretical analysis of experimental data, as well as by the theoretical or semiempirical predictions.

As a fragmentation process of parent nucleus in two unequal nuclei, the cluster radioactive decay is an intermediate phenomenon between the symmetric fission and alpha decay. Obviously, the cluster radioactivity can be approached theoretically from two viewpoints: of the fission theory and of alpha-decay theory. Until now, the first one was extensively used in calculations.

Both treatments have in common the calculation of penetrability factor of the two fragments in the decay channel through the Coulomb (or Coulomb plus nuclear) barrier. The penetrability factor is very sensible to the Q value

of the decay and to the barrier dimensions, offering the gross feature of decay probabilities. The simplified versions of cluster decay models, like [1], or more elaborated and recent ones, like [4] and [9] are based mainly on the calculation of penetrability. The nuclear structure was used there to obtain the Q-values of cluster decay, or the relative l-value for the centrifugal term of the penetrability factor. This simple treatment, build on fission models, is in fact a semiempirical one. Nevertheless, it contributed decisively to the present progress in this new field of Nuclear Physics.

The situation changed with the discovery of fine structure of ^{223}Ra , which decays emitting a ^{14}C cluster, on ground and excited states of ^{209}Pb , ref. [7]. The theory has to explain now the hindrance factors which are directly dependent on the microscopic structure of the parent, daughter and emitted nuclei [8].

In the fission-like approach of cluster radioactive decay (see, for example the analytical superasymmetric fission model, ASAFM, ref. [9]) is principally difficult to introduce the detailed nuclear structure, as the shell model configurations. To describe the hindered decays within ASAFM, the authors of ref. [10] had to modify the potential barrier in the decay channel following a semiempirical procedure. Even the cluster preformation was described as a barrier penetrability [22]. In this way, the prediction of hindrance factors becomes difficult.

Several attempts to calculate the absolute α - *decay* probabilities have been made. (See for a review, refs. [16] and [11]). One of the aims was

to obtain a stability of the absolute decay probabilities against a reasonable variation of the channel radius, R_0 and of the parameters of $V_{A-4,\alpha}(R)$, the interaction potential between the fragments in the final channel.

Following a non R-matrix theory developped in ref. [13], the authors of ref. [14] calculated the absolute α -decay widths of spherical nuclei in the framework of Woods-Saxon shell model, proposing also new criteria for the classification of alpha decays. These calculations were extended to study the effects of configuration mixing, ref. [15] and of superfluidity, ref. [16]. The problem of relative α -decay probabilities was also examined in connexion with the high spin isomers in Polonium region [15].

Two important attempts to generalize the alpha-decay theories for the cluster radioactive decay were performed. The first one, ref. [23], based on the R-matrix theory, used approximative methods, like the development of WS wave functions in HO series, without a numerical estimation of the errors. The second one, ref. [18] is based on the so called cluster approach to alpha decay, ref. [16]. The cluster width calculated in ref. [18] plays the role of a spectroscopic factor for the cluster radioactive decay.

In fact, the problem of obtaining the theoretical values of cluster radioactive decay probabilities remains open. As a first step towards reaching this goal, the present paper describes a suitable method to calculate the overlap integral of cluster decay in the framework of WS shell model.

2 The overlap integral in α -decay

In the α -decay theory, the overlap integral is, by definition, the following expression:

$$\begin{aligned}
 I_\alpha^L(R) &= \langle \Phi^{I_i M_i} | a \Phi_C^{I_i M_i} \rangle \\
 &= \langle \Phi^{I_i M_i}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A) | \\
 &\quad a \{ [\Phi^{I_f}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_{A-4}) \Psi^L(\vec{R})]^{I_i M_i} \chi_\alpha(\vec{\xi}_1, \vec{\xi}_2, \vec{\xi}_3) \} \rangle
 \end{aligned} \tag{1}$$

where $\Phi^{I_i M_i}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A)$ and $\Phi^{I_f M_f}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_{A-4})$ are the wave functions of the initial and final nuclei. χ_α is the α -particle internal wave function, depending on the internal Jacobi coordinates $\vec{\xi}_1, \vec{\xi}_2, \vec{\xi}_3$. The function $\Psi^L(\vec{R})$ is the solution of the Schroedinger equation for the relative motion between the daughter nucleus and the alpha-particle and can be written as the product between the radial $\Phi^L(\epsilon, R)$ and angular $Y_\mu^L(\Omega_R)$. \vec{R} is the distance between the daughter nucleus and the emitted alpha-particle and ϵ is the kinetic energy of their relative motion. The operator a stands for the antisymmetrization in the final channel represented by the wave function $\Phi_C^{I_i M_i}$.

The main difficulty to calculate the overlap integral is to separate the centre of mass (COM) motion for the four nucleons of the parent nucleus entering the alpha-particle. When the wave functions of the initial and final nuclei are given in the HO shell model, this separation can be performed by using Mang's formula for the ground state to ground state transitions, (ref. [12]) or by performing three Talmi-Moshinsky transformations [19] to pass

from the coordinates \vec{r}_i ($i=1,2,3,4$) to \vec{R} and $\vec{\xi}_j$ ($j=1,2,3$).

This problem becomes a principal one when aiming to obtain the absolute theoretical decay widths. Theoretical considerations, like the Pauli principle, as well as the overlap integral calculations show that the most important contribution for the alpha cluster preformation probability and hence for the absolute decay probability comes from the surface region of parent nucleus [14]. Or, in this region, the correct behaviour of one particle wave functions is given by the Woods-Saxon shell model. As a consequence, WS shell model must be used in the calculation of absolute α -decay probabilities instead of the HO ones.

The calculation of the overlap integral within the WS shell model is currently done by developing the WS wave functions in series of HO functions, which allows the subsequent use of the standard Talmi-Moshinsky technique. However, this procedure increases drastically the number of matrix elements to be calculated and rises complicated problems in connection with the convergence of HO series.

The problem of separating the COM motion for WS wave functions was solved in ref. [17] and applied in refs. [14] and [15] to the calculation of α -decay probabilities. When passing from HO shell model to WS one, the absolute alpha-decay widths Γ_α decrease by a factor of 6 to 27, which depends on the shell model configurations of the four nucleons preforming the alpha

particle. This result, obtained in refs. [14] and [15] points to the fact that even the relative alpha-decay probabilities can be seriously affected when passing from HO to WS shell model wave functions. These findings are also important for calculating the absolute cluster radioactive decay probabilities.

3 The cluster overlap integral

The cluster overlap integral for an initial (parent) nucleus of mass A ,

$$\Phi_A^{I_i M_i}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A)$$

decaying to a final (daughter) nucleus

$$\Phi_{A-A_c}^{I_f M_f}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_{A-A_c})$$

by emitting a cluster of mass A_c and having the internal wave function

$$\Phi_{A_c}^{I_c M_c}(\vec{\xi}_1, \vec{\xi}_2, \dots, \vec{\xi}_{A_c-1})$$

is given by the formula:

$$I_{A, A_c}^{I_i I_f L}(R) = \langle \Phi_A^{I_i M_i} | a \{ [\Phi_{A-A_c}^{I_f} \Phi_{A_c}^{I_c} Y^L(\Omega_{\vec{R}})]^{I_i M_i} \Psi_c^L(R) \} \rangle \quad (2)$$

Here $\Psi_c^L(R) Y_\mu^L(\Omega_{\vec{R}})$ is the cluster external wave function, which is the solution of the Schroedinger equation for the relative motion between the daughter nucleus and the emitted cluster; \vec{R} is the relative distance between them. This formula is a natural extension of the α -decay overlap integral, being also similar to the R-matrix formula given in ref. [23] and to the cluster approach

formula from ref. [18].

The approach of ref. [18] has the advantage of using shell model functions given directly in the Jacobi coordinates. This fact makes the calculation much easier, but such a procedure is restricted to HO shell model only. The HO shell model can be, eventually, used for the cluster internal wave function, when A_c is small, but not for the wave functions of parent and daughter nuclei. In the references [23] and [18] approximative methods of calculation were used. In the first one, the Moshinsky brackets for different masses of particles in HO potential well [20, 21] were approximated with the ordinary Moshinsky brackets. In the ref. [18] the HO constants of the parent, daughter and emitted nuclei were taken equal, and a correction factor was introduced. These two approximations are in fact equivalent.

As it was shown in refs. [14, 15] the Woods-Saxon shell model is more appropriate to calculate the α -decay overlap integral. This must be true also for the cluster radioactive decay. The number of nucleons forming the emitted particle in the cluster radioactive decay is much greater than the analogue number in the alpha-decay, $A_c \gg 4$, consequently, one expects there a greater difference between WS and HO shell model calculations than the factor of 6 to 27 found in the ref. [14] for the case of α -decay.

Unfortunately, the previous attempts to find a correction formula allowing for the calculation of WS shell model α -decay absolute probabilities starting from the HO ones had failed (see ref. [14]). This conjecture holds undoubtedly also for the cluster emission, where more requirements are imposed to

such a formula. So, the calculation of absolute decay probabilities in the framework of WS shell model becomes very important for understanding the limitations of the theory and for evaluating the different approximations used in this field.

One of these approximations is the development of the WS shell model wave functions in HO series, ref. [23]. However, such a development in series needs a careful study of convergence and an optimal cutoff. The HO and WS wave functions have different behaviour at the nuclear surface and in the asymptotic region and the convergence reached for one region doesn't extend automatically to the other. To reach a better global convergence, a greater number of terms must be retained. If, after the cutoff, the minimum number of terms left in each series is n , then the total number N of the overlap integrals $I_{A,A_c}^{I_i I_f L}(R)$ to be calculated becomes greater than n^{A_c} . Even for the smallest possible value of n , ($n=2$), the number of the overlap integrals to be calculated for the (yet hypothetical) emission of the ^{40}Ca cluster becomes greater than 10^{12} . The calculation of ^{40}Ca radioactivity plays a test role for the decay models; the emission of such a cluster is very near to the region of the asymmetric fission. In the ideal case, the theoretical description of phenomena must pass smoothly from the alpha decay, to the cluster emission and fission. (A detailed discussion of this subject can be found in ref. [18]). For (a reasonable small) $n=5$, N is greater than $6 * 10^9$ for ^{14}C emission and than $2 * 10^{22}$ for ^{32}Si emission. These numbers increase considerably if the configuration mixing takes place in the initial and (or) final states of nuclei. The above evaluation shows that the expansion of WS wave functions in HO series cannot be used to calculate the absolute decay probabilities, except

maybe the simplest cases of cluster emission in single particle shell model. Even for the smallest A_c and for a minimum of terms in the HO series, the repeated application of the Talmi-Moshinsky transformation is confronted with difficulties. The ordinary Moshinsky brackets, ref. [19] can be used only when the masses (or the reduced masses) of the involved particles (or group of particles) are equal at each step of the transformation. (Another equivalent condition is the equality of oscillator constants.) The only emitted cluster which fullfils this condition is ^{32}Si as having A_c equal to a power of 2, allowing a symmetrical graph of the Talmi-Moshinsky transformations. For all the rest of emitted clusters, at a certain step, a special kind of Moshinsky bracket is needed (for unequal masses). The analytical formula for this type of Moshinsky bracket, ref. [20], is much more complicated than the usual one. Another formula, see ref. [21], gives the special Moshinsky bracket as a sum of products of usual Moshinsky brackets.

A way to circumvent these difficulties is to follow the method of separating the COM motion for two particles in a WS potential well, developed in ref. [17] and adapted in ref. [14] for absolute alpha-decay calculations.

After the usual separation of two particles wave functions through the fractional parentage coefficients, the typical integral to be calculated in the formula (2) has the form:

$$A_{ik}^{LMlm}(\vec{R}_{ik}) = \int [\phi_{n_i l_i}(\vec{r}_i) \phi_{n_k l_k}(\vec{r}_k)]_{LM}^* \chi_{lm}(\vec{r}_{ik}) d\vec{r}_{ik} \quad (3)$$

where $\phi_{n_{i(k)} l_{i(k)}}(\vec{r}_{ik})$ are the WS wave functions of two nucleons from the parent nucleus, labeled $i(k)$, entering the emitted cluster, after the separation of the spin variables. $\chi_{lm}(\vec{r}_{ik})$ is the part of the cluster internal wave function

containing the relative coordinate of nucleons i and k , $\vec{r}_{ik} = \vec{r}_i - \vec{r}_k$. $A_{ik}^{LMlm}(\vec{R}_{ik})$ represents the wave function of COM motion of particles i and k , $\vec{R}_{ik} = \frac{m_i\vec{r}_i + m_k\vec{r}_k}{M}$ being the COM coordinate. M stands for $m_i + m_k$. Here, for convenience, we took different masses for the two nucleons. After a change of coordinates, ref. [17] and an integration over the angular variable $\phi_{\vec{r}_{ik}}$, we obtain:

$$A_{ik}^{LMlm}(\vec{R}_{ik}) = \sum_J Y_{J,M-m}(\Omega_{\vec{R}}) C(LLJ; M-m)(-1)^m B_{ik}^{JLl}(R_{ik}) \quad (4)$$

where

$$B_{ik}^{JLl}(R_{ik}) = \frac{1}{2} \sqrt{\frac{(2l_i+1)(2l_k+1)(2l+1)}{2J+1}} \sum_{pq} (-1)^{p+q} C(LLJ; p+q-p-q) C(l_i l_k L; pq) \int \phi_{n_i l_i}(r_i) \phi_{n_k l_k}(r_k) P_{l_i}^p(\cos \theta_i) P_{l_k}^q(\cos \theta_k) P_l^{p+q}(\cos \theta_{ik}) r_{ik}^2 dr_{ik} d(\cos \theta_{ik}) \quad (5)$$

The following notations were used:

$$r_i = \sqrt{R_{ik}^2 + \frac{m_k^2}{M^2} r_{ik}^2 + 2 \frac{m_k}{M} R_{ik} r_{ik} \cos \theta_{r_{ik}}} \quad (6)$$

$$r_k = \sqrt{R_{ik}^2 + \frac{m_i^2}{M^2} r_{ik}^2 - 2 \frac{m_i}{M} R_{ik} r_{ik} \cos \theta_{r_{ik}}} \quad (7)$$

$$\theta_i = \arcsin\left(\frac{\frac{m_k}{M} \sin \theta_{r_{ik}}}{r_i}\right) \quad (8)$$

$$\theta_k = \arcsin\left(\frac{\frac{m_i}{M} \sin \theta_{r_{ik}}}{r_k}\right) \quad (9)$$

Here $C(l_1 l_2 L; m_1 m_2)$ are the Clebsh Gordan coefficients and $P_l^m(\cos \theta)$ are the Legendre polynomials. The double integral in the formula (5) is performed using the Gauss methods. By replacing there the WS wave functions

with HO ones and putting $m_1 = m_2$, the formula (5) becomes a usual Talmi-Moshinsky formula for the separation of COM. The same formula gives a COM separation also for two particles of different mass. In both cases, the calculations were confronted with the previous results, ref. [19], where the Moshinsky brackets are given in the tables, or refs. [20] and [21]. The comparison of computing times of the classical multiple summations from the last three references with the formulae (4) and (5) shows a clear advantage of the method presented here. The actual method requires the same time for equal masses of particles, or for different ones.

To calculate the WS shell model overlap integral for cluster emission, the formula (4) must be applied repeatedly. For the ^{14}C emission, for which the first trial calculation was done, the COM separation implies the application of the above procedure seven times for pairs of nucleons, three times for groups of four nucleons and once for a group of eight nucleons, in all cases the masses of particles or the reduced masses of the components of groups being equal. The last two COM separation procedures implying a group of six nucleons (4+2) and 14 nucleons (8+6) correspond to the transformation of coordinates for different reduced masses. At this point, the authors of ref. [23] used the ordinary Moshinsky matrices instead of the special ones, ref. [20] or [21].

The simplification brought by the formula (5) allows the correct calculation of the cluster decay overlap integral in the WS scheme, opening in this way the possibility to obtain the theoretical decay probabilities.

4 Conclusions

The present paper proposes a method of calculating the overlap integral for the cluster radioactive decay, in the framework of WS shell model, suitable for computation. The same method can be used for other approaches, using for example HO basis. The problem of obtaining the absolute theoretical values in the cluster radioactive decay needs further developments, which are very important to evaluate different approximations currently made.

As it is well known, even in a simpler case like the alpha-decay, the theoretical approaches are meeting difficulties in describing the absolute values. Nevertheless, many standard procedures of calculation and some classification criteria were developed in the alpha-decay theory; their application to the cluster radioactivity could improve our knowledge in this new field.

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