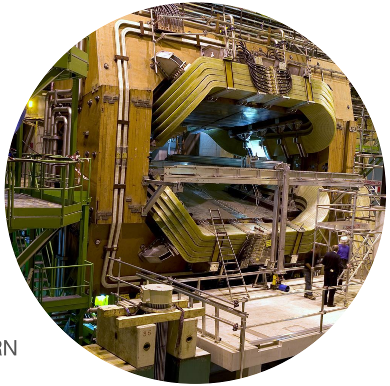
Vertex Reconstruction Using the General Triplet Track Fit for the Mu3e Experiment

Thomas Schwartze
Physikalisches Institut, Universität Heidelberg

19.06.2026 - CLFV 2026 Rome
Best Experimental Poster Award Talk

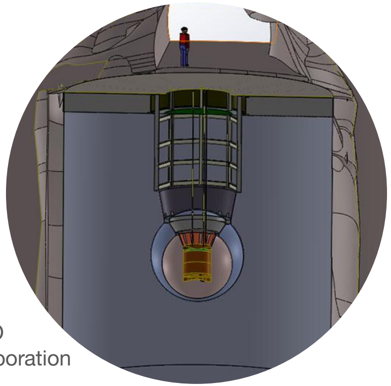
What do these experiments have in common?

LHCb



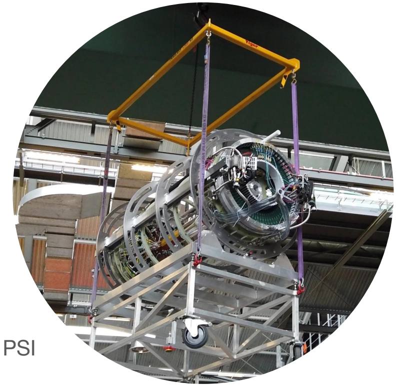
CERN

nEXO



nEXO
Collaboration

Mu3e



PSI

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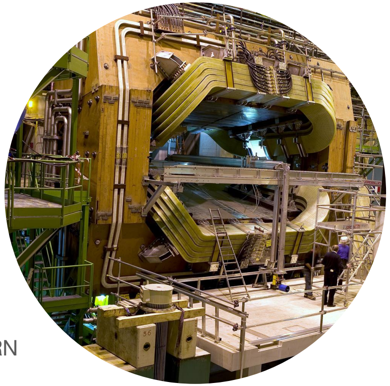

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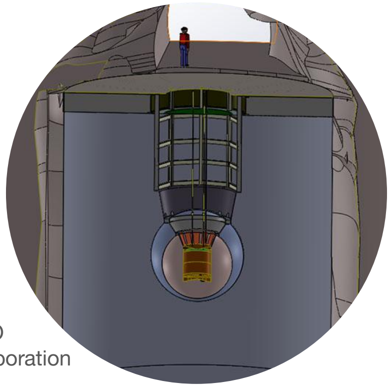

nEXO Collaboration

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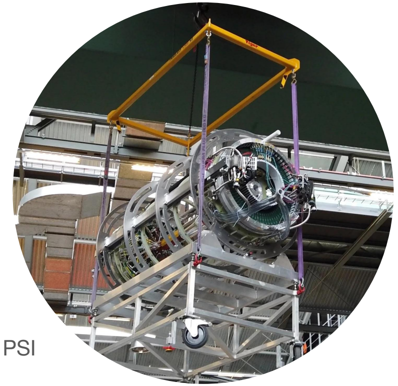

PSI

Vertex reconstruction!

What do these experiments have in common?

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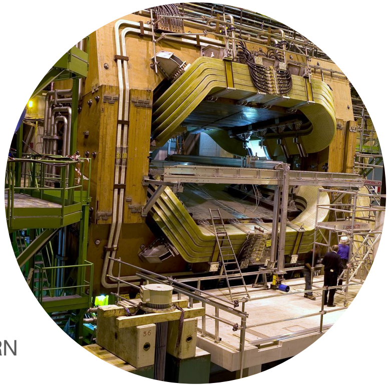
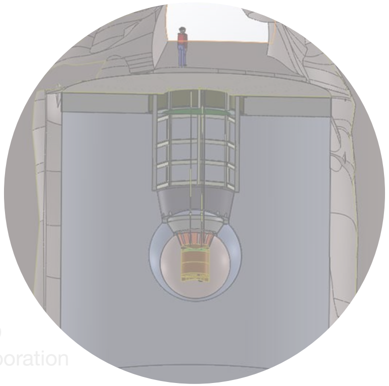


CERN

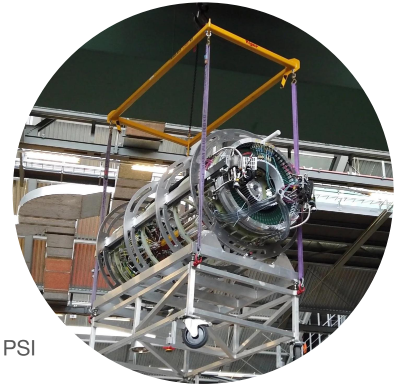
High-Luminosity LHC

nEXO



nEXO Collaboration

Mu3e



PSI

High-Intensity Muon Beam

Precise vertex reconstruction!

Essential in high-luminosity / high-intensity experiments with many overlapping interactions

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Vertex Reconstruction

Current Standard: **Kalman Filter**

Input: **fitted tracks**

⬇

Extrapolation of tracks to estimate the vertex position

⬇

Update of vertex position, Iterate until position converges

Highly successful, but sequential

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Current Standard: **Kalman Filter**

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My master project: **Vertex Fit Using the General Triplet Track Fit**

Input: **detector hits**

⬇

Simultaneous fit of track parameters and vertex position

⬇

Analytical solution for the vertex position

Parallelizable

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How precise is the GTTF vertex reconstruction?

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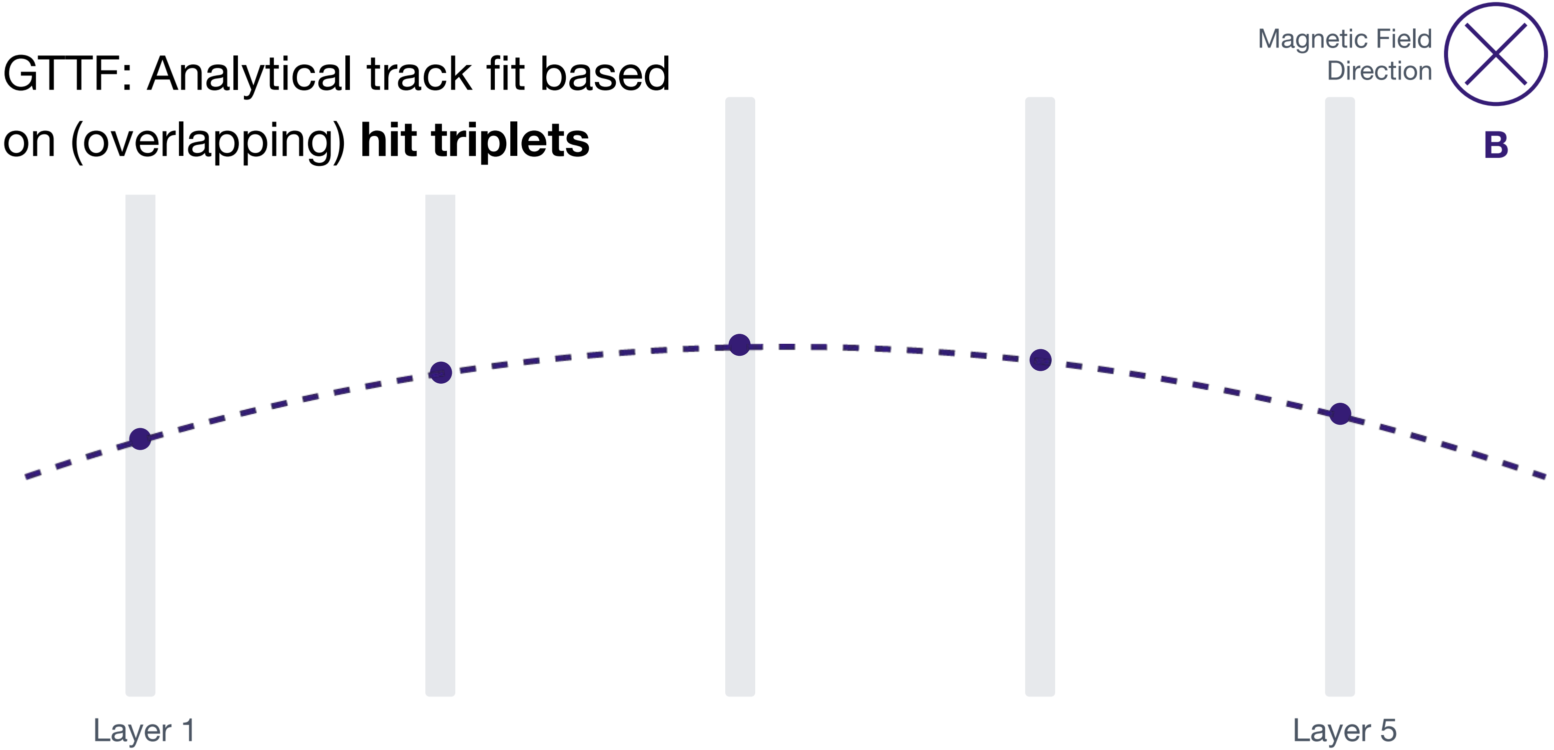
General Triplet Track Fit (GTTF)

- GTTF: Analytical track fit based on (overlapping) **hit triplets**



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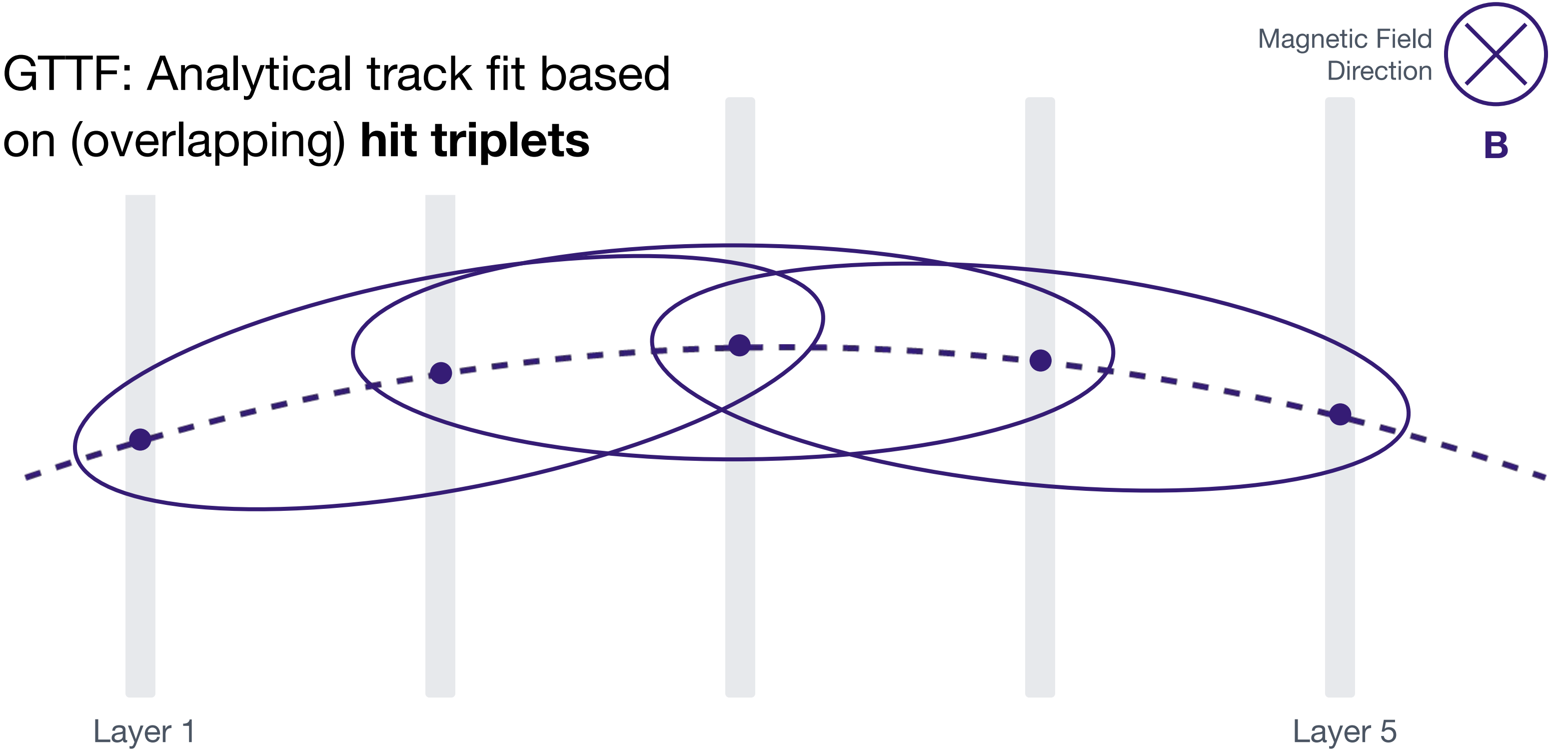

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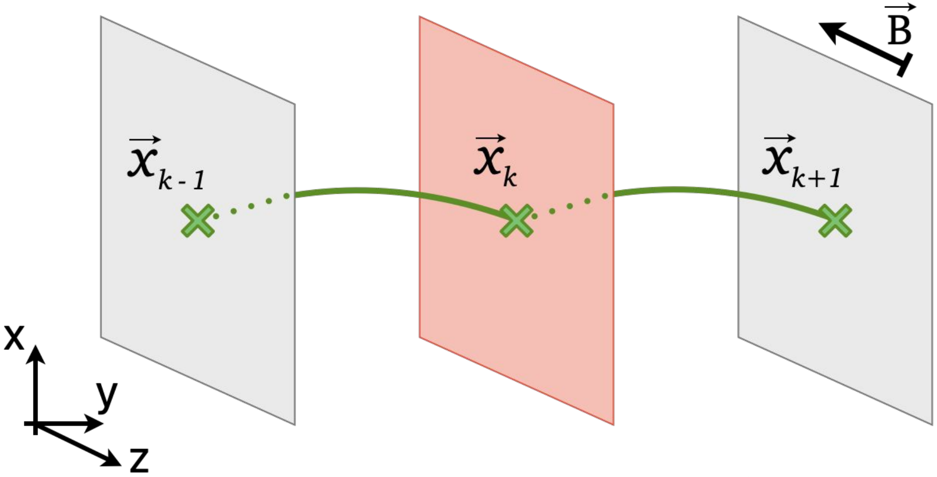

Adapted from D. Fritz, MSc. Thesis

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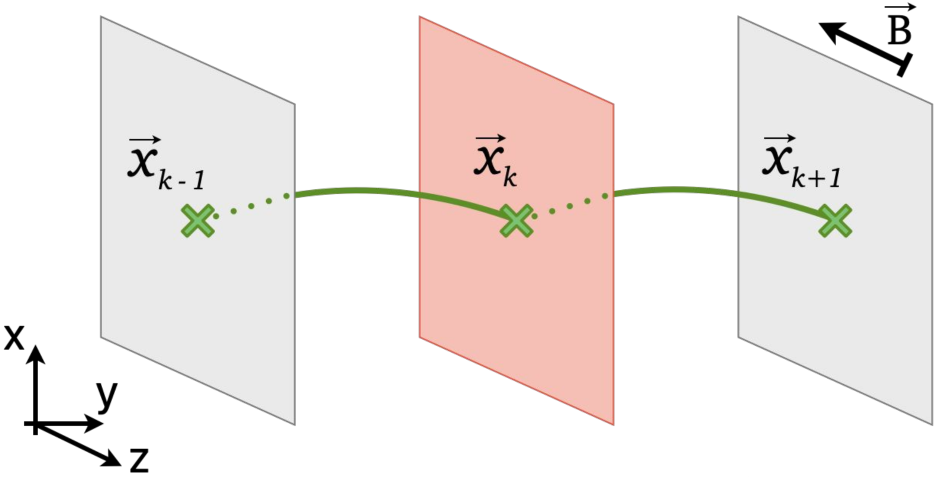
- Fit of the **curvature** and **hit positions**



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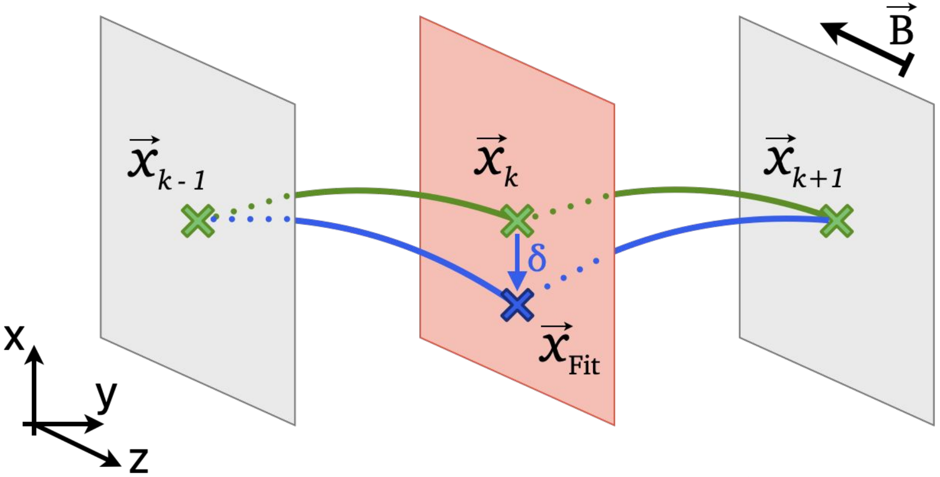

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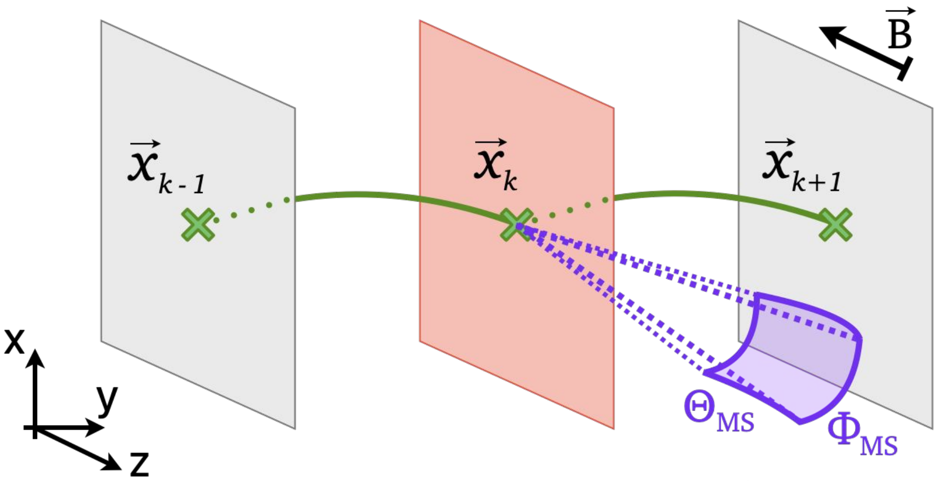

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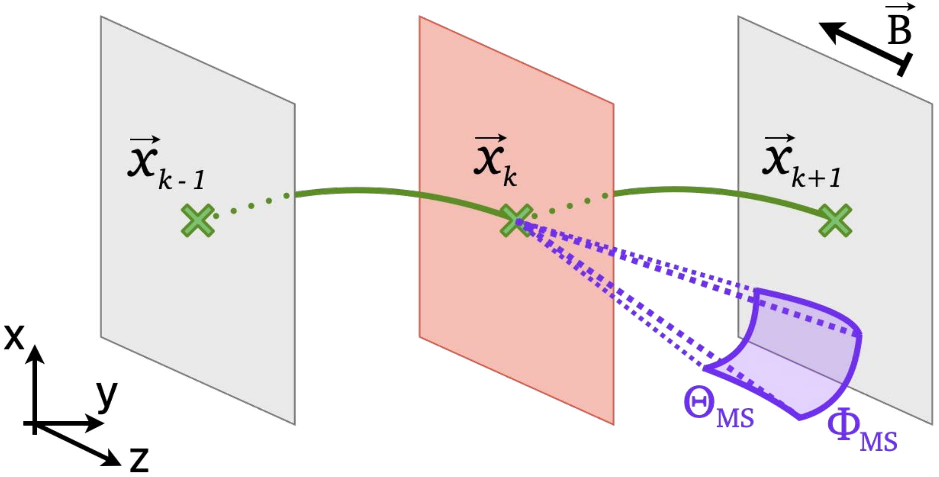

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$$\chi^2 = \boxed{\sum_{j=0}^{n_{\text{scatt}}-1} \left(\frac{\Theta^2_{MS,j}}{\sigma^2_{\Theta_{MS,j}}} + \frac{\Phi^2_{MS,j}}{\sigma^2_{\Phi_{MS,j}}} \right)} + \boxed{\sum_{k=0}^{n_{\text{hit}}-1} \vec{\delta x}_k^{\top} \vec{\vec{V}}_k^{-1} \vec{\delta x}_k}$$

Multiple scattering | Hit uncertainty

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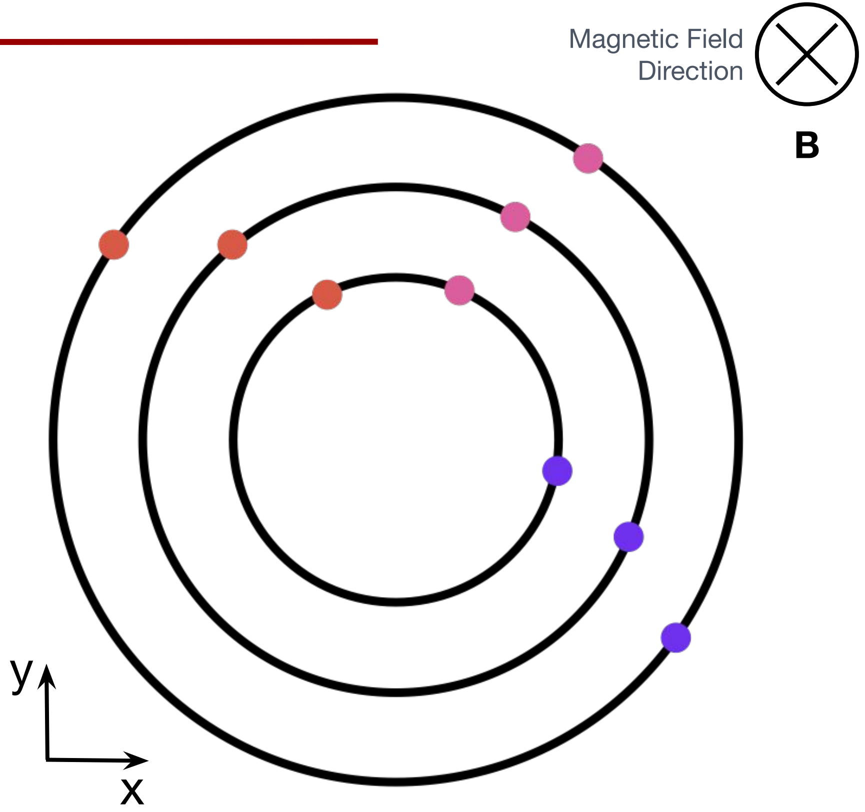
GTTF: Vertex Fit

Step 1: Track reconstruction

- ❑ Track finding

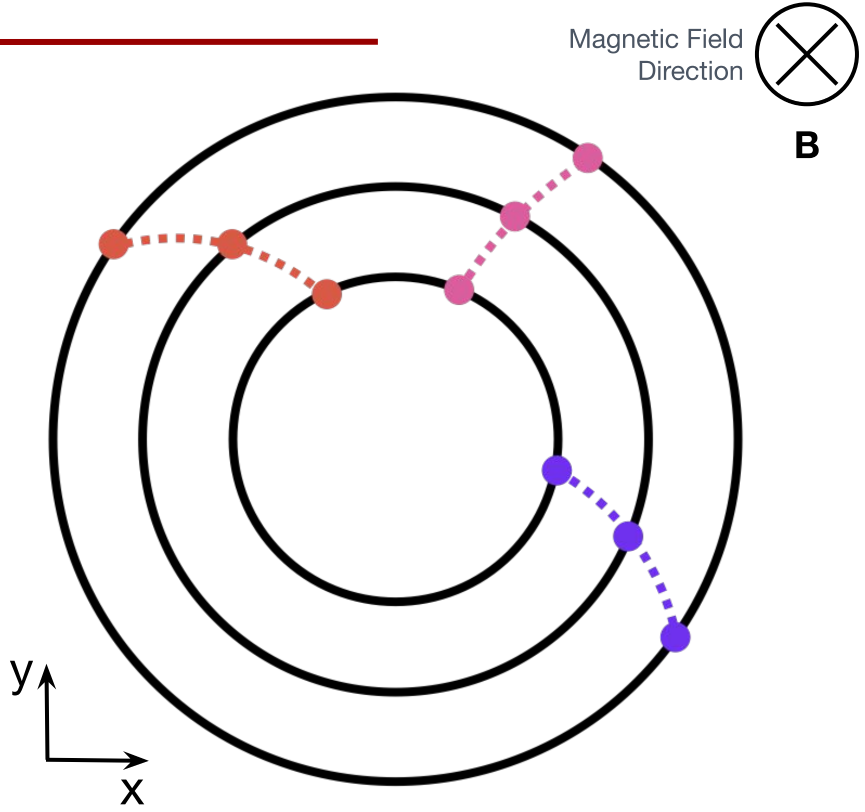


Example for a three-prong decay

GTTF: Vertex Fit

Step 1: Track reconstruction

- ❏ Track finding
- ❏ Track fitting



Example for a three-prong decay

GTTF: Vertex Fit

Step 1: Track reconstruction
- ❏ Track finding
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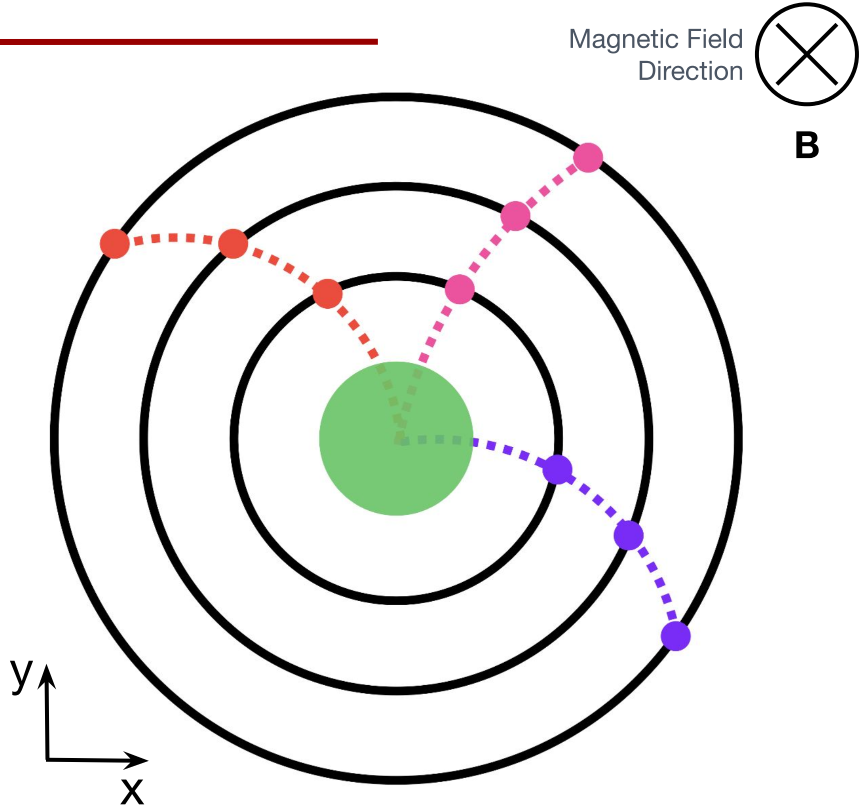
Step 2: Track extrapolation
- ❏ Find initial vertex guess



Example for a three-prong decay

GTTF: Vertex Fit

Step 1: Track reconstruction

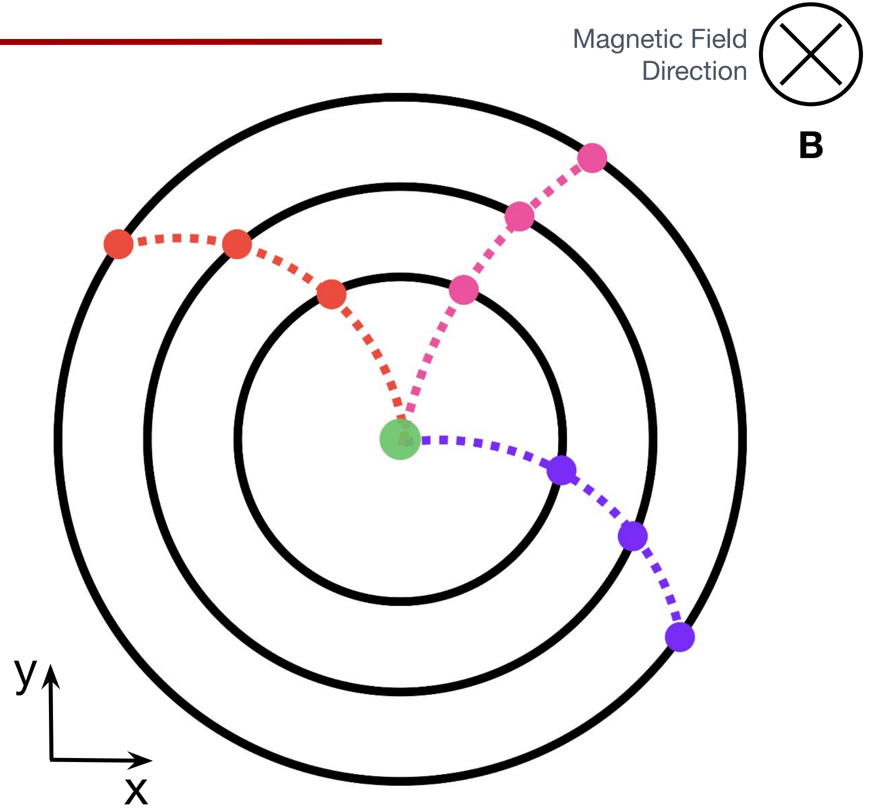
- ❑ Track finding
- ❑ Track fitting

Step 2: Track extrapolation

- ❑ Find initial vertex guess

Step 3: Vertex Fit

- ❑ Vertex position = pseudo-hit with large position uncertainty



Example for a three-prong decay

GTTF: Vertex Fit

Step 1: Track reconstruction

- ❏ Track finding
- ❏ Track fitting

Step 2: Track extrapolation

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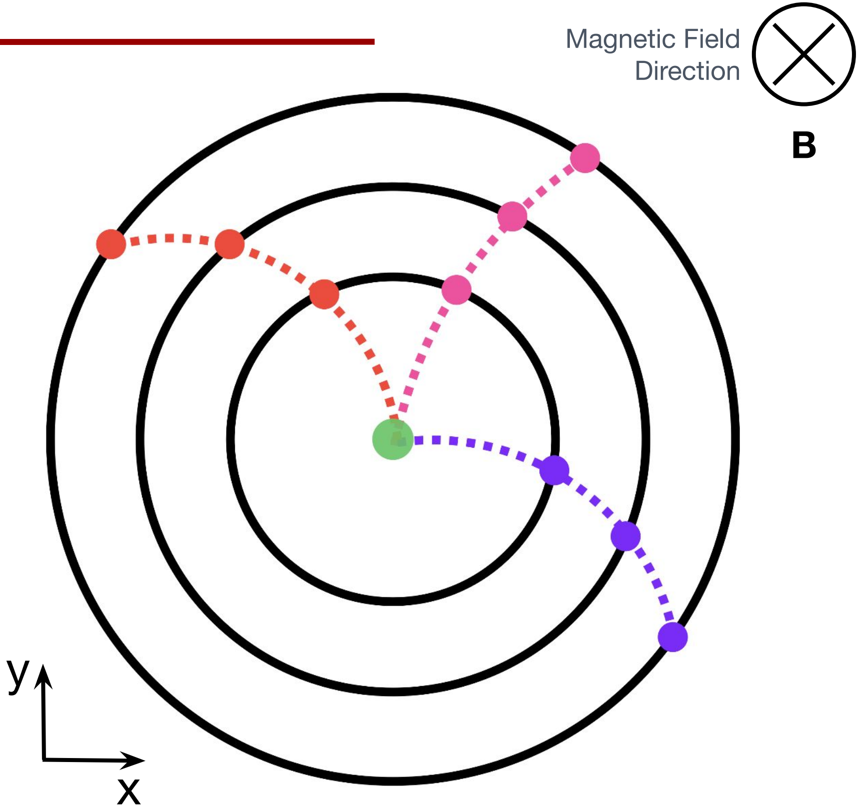
$$\chi^2_{\text{Vertex Fit}} = \sum_{i=1}^{N_{\text{tracks}}} \chi_i^2 + \vec{\delta}^T_{\text{vtx}} \mathrm{V}_{\text{vtx}} \vec{\delta}_{\text{vtx}}$$



Example for a three-prong decay

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Vertex Reconstruction Using the General Triplet Track Fit for th
Mu3e Exp

for a Toy Detector

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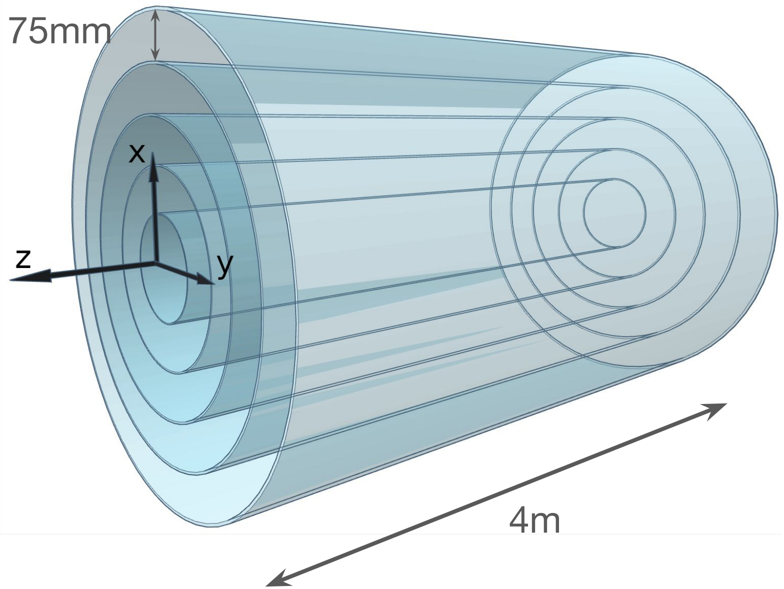
Toy Detector Validation

Detector geometry:

- ❑ 5 Barrel layers
- ❑ First layer: r = 40mm
- ❑ Uniform magnetic field with B = 1T in z-direction



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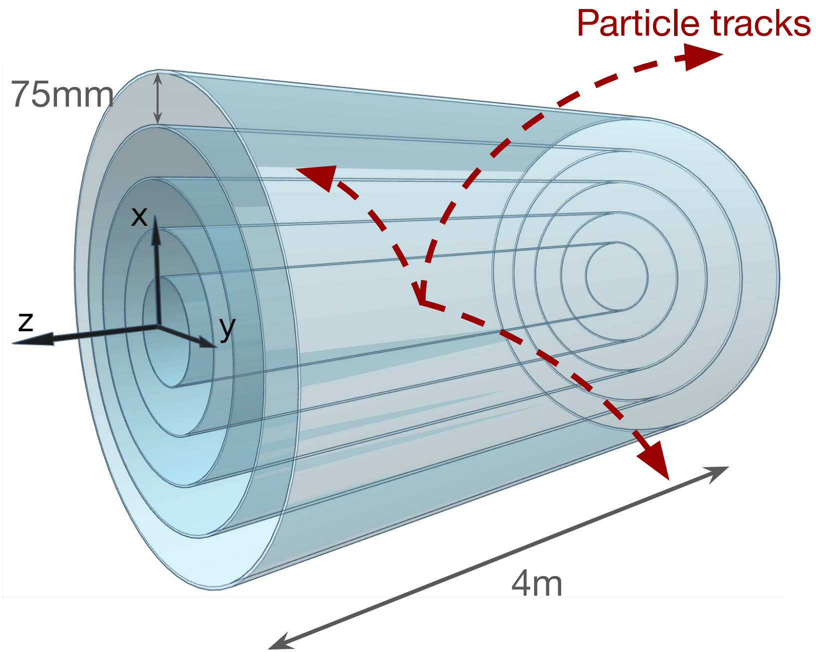
Event generation:

- ❑ 3 particles/event at a common vertex
- ❑ Fixed momentum for all three particles
- ❑ Isotropic in ϕ
- ❑ $\Theta \in [-1.2, 1.2]$ rad
- ❑ Vertex position: Gaussian distributed along z



Toy Detector Validation

Vertex fit quality χ^2

Vertex position

Output of Vertex Fit

Toy Detector Validation

Vertex fit quality χ^2

Default Detector:

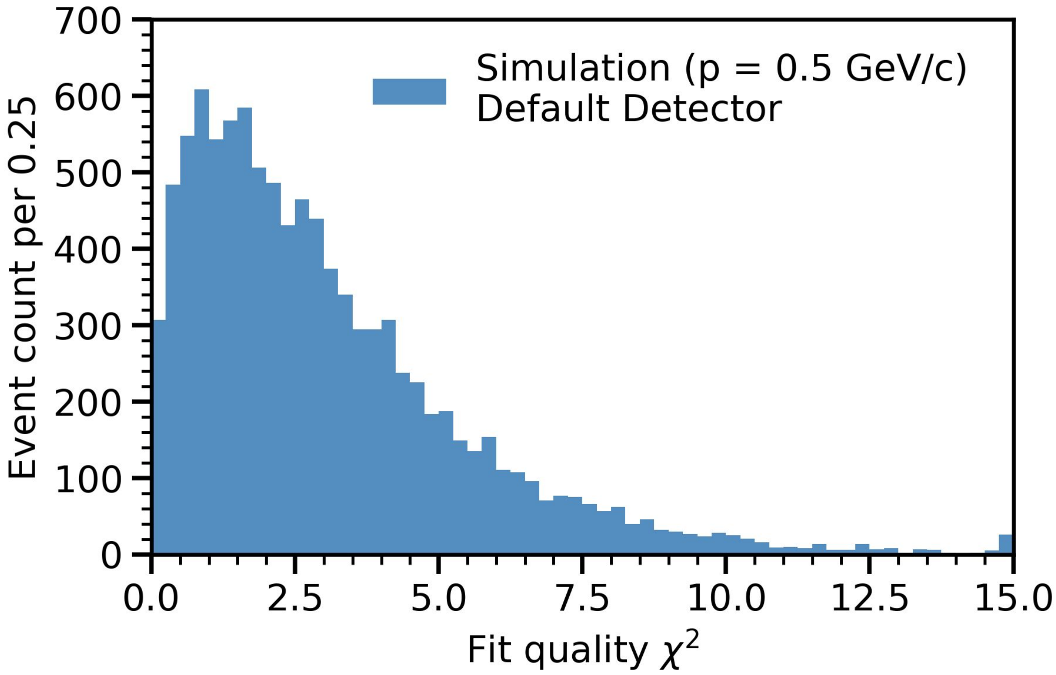
- ❏ Material per layer: 0.002 X/X_0
- ❏ Sensor size: [50 µm, 50 µm]

Generation:

- ❏ p = 0.5 GeV/c

Toy Detector Validation

Vertex fit quality χ^2



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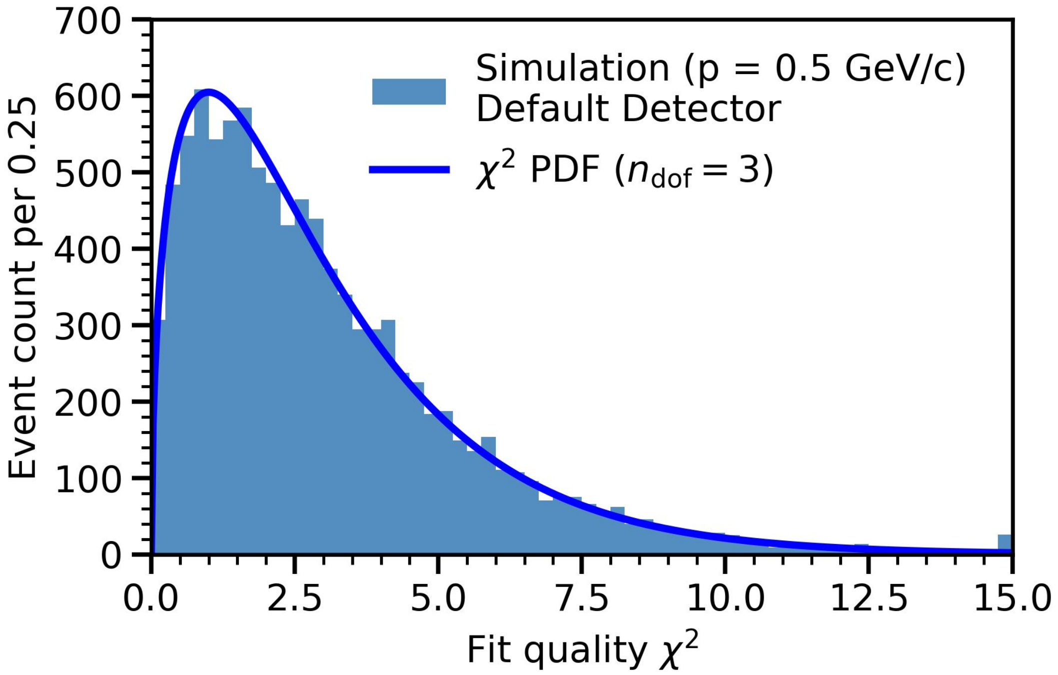
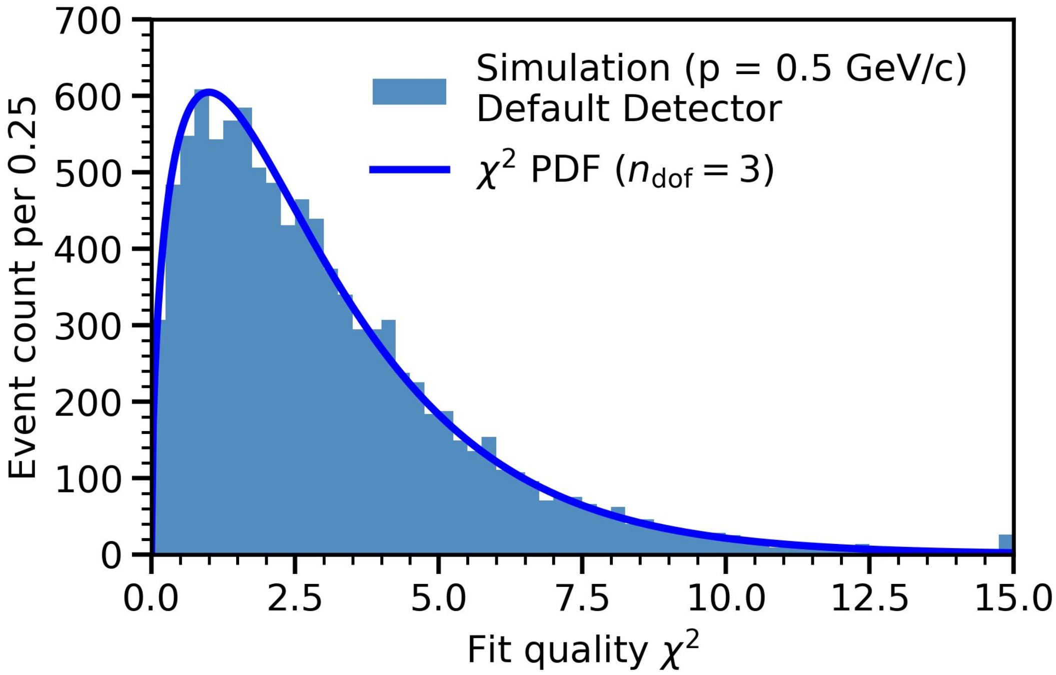


Toy Detector Validation

Vertex fit quality χ^2 ✅



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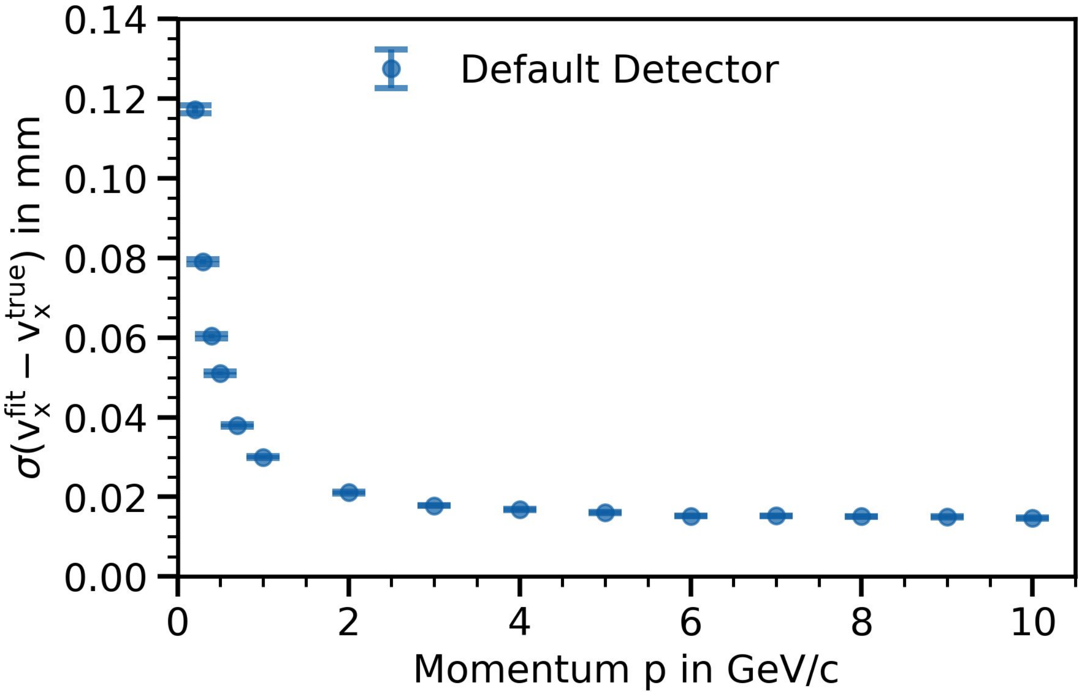

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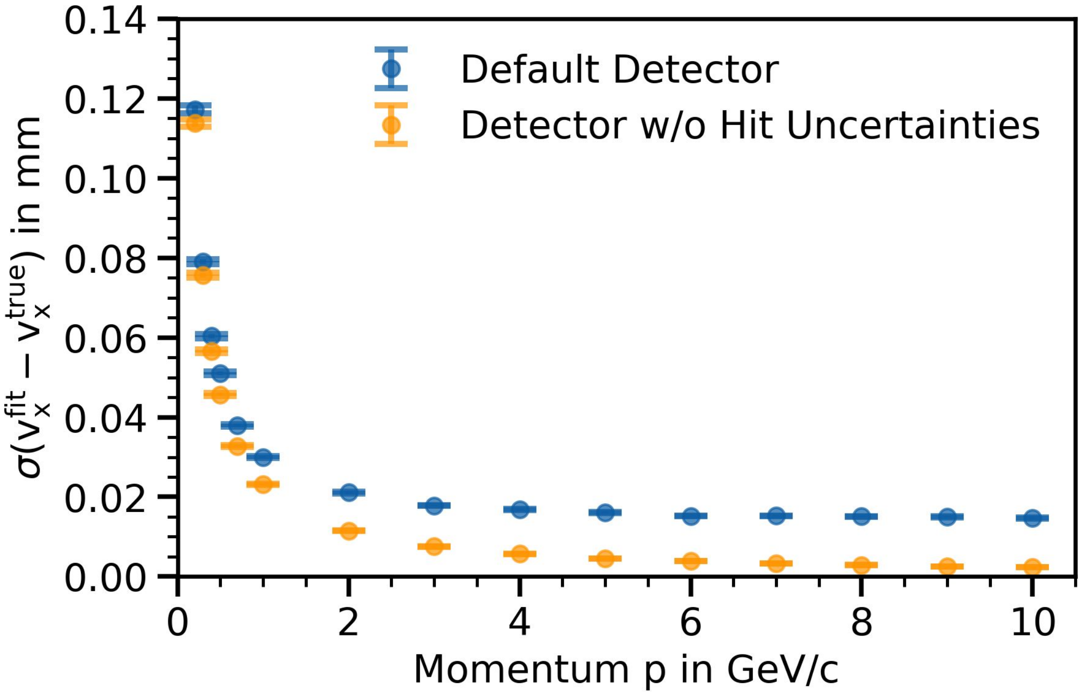
Detector w/o Hit Uncertainties:

- ❑ Material per layer: 0.002 X/X_0
- ❑ Sensor size: ~ [0 µm, 0 µm]



Toy Detector Validation

Vertex position

Default Detector:

- Material per layer: 0.002 X/X_0
- Sensor size: [50 µm, 50 µm]

Detector w/o Hit Uncertainties:

- Material per layer: 0.002 X/X_0
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Detector w/o Material:

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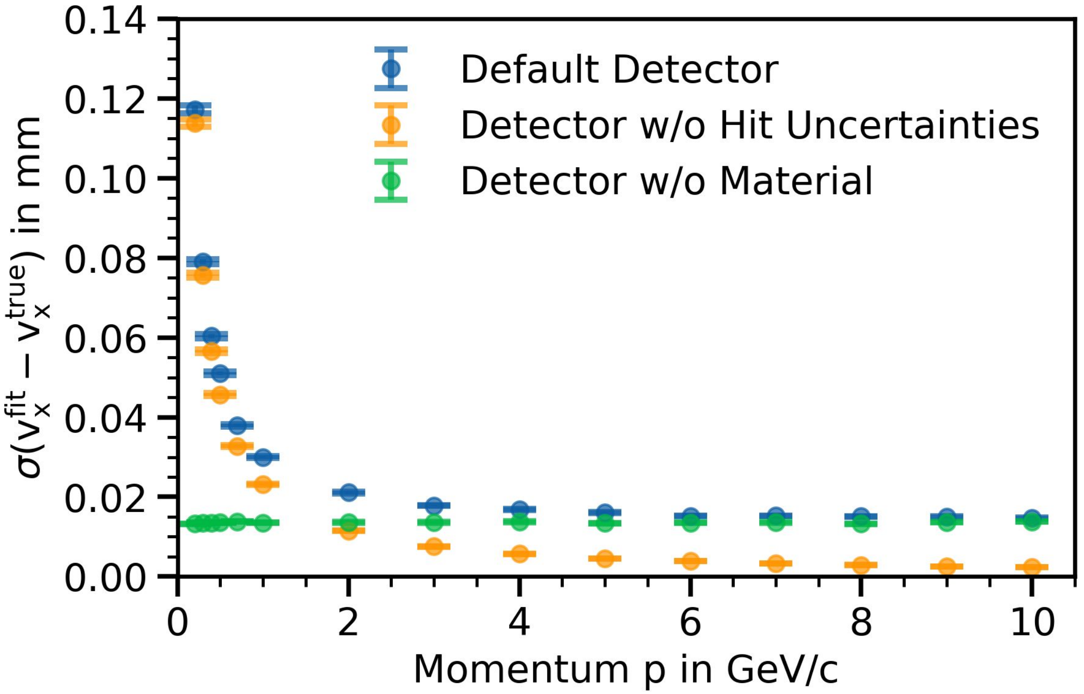
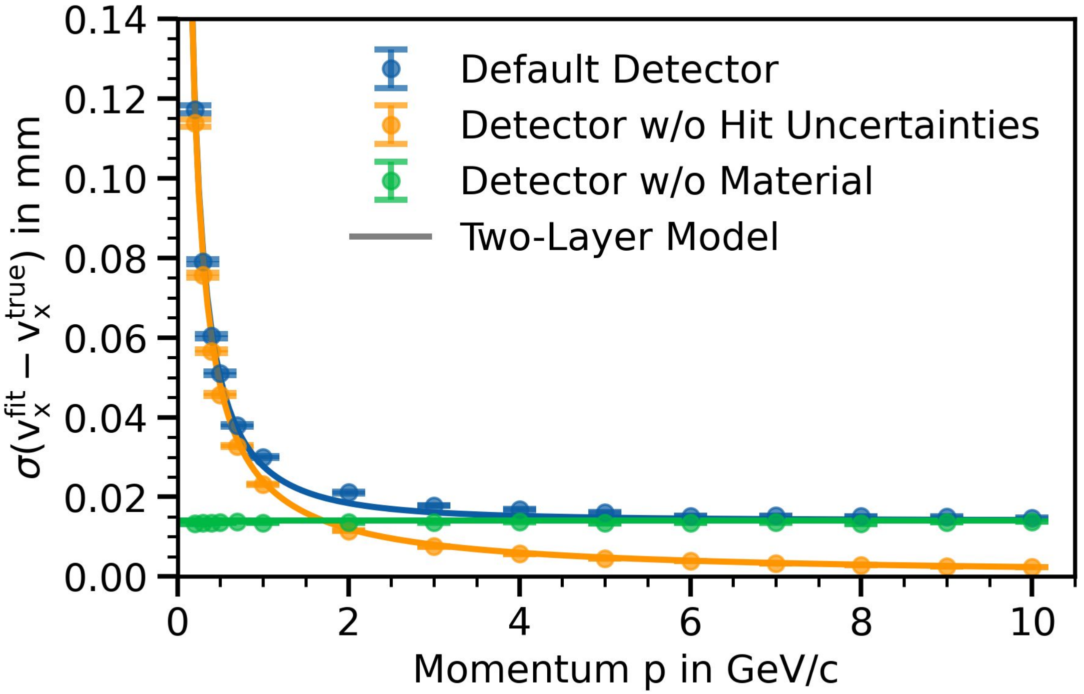


Toy Detector Validation

Vertex position

Two-Layer Model:

- Theoretical calculation of resolution for a model with only two tracking layers
- Achievable resolution given by the **detector geometry** & **particle momentum**



Toy Detector Validation

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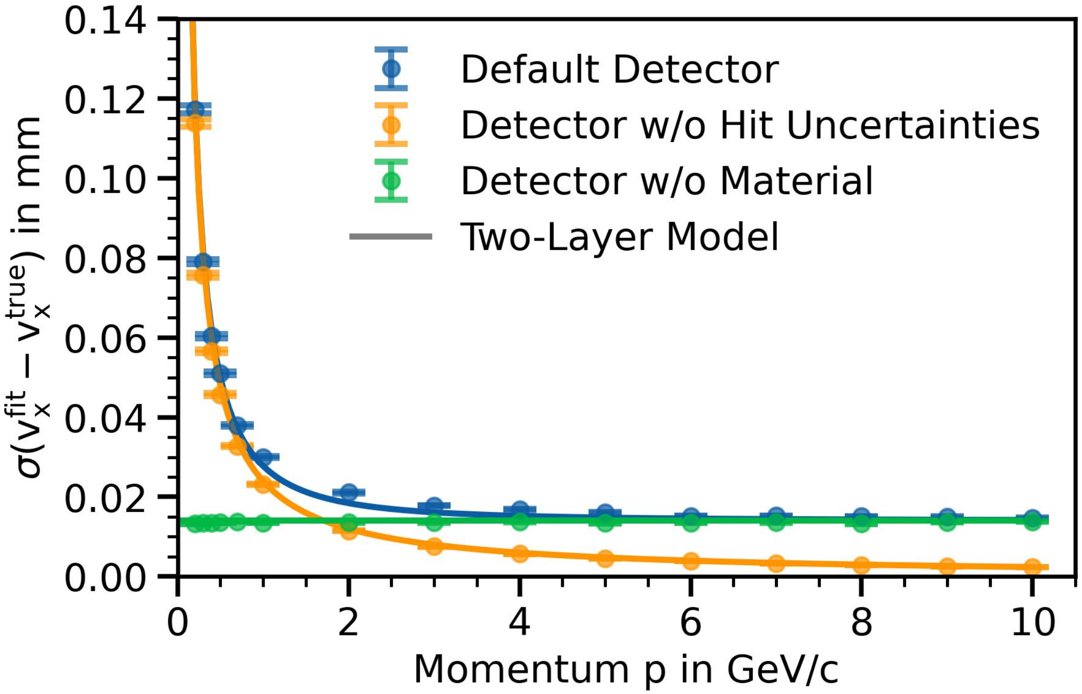

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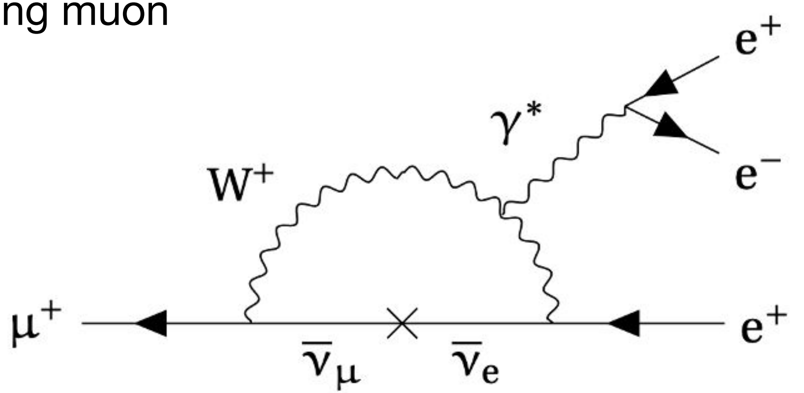
Mu3e Experiment: Motivation

- Search for the Charged Lepton Flavour Violating muon decay: $\boxed{\mu^+ \to e^+ e^- e^+}$

- Highly suppressed in the Standard Model + Neutrino Mixing:

$$\mathrm{BR}(\mu^+ \to e^+ e^- e^+) \approx 10^{-54}$$

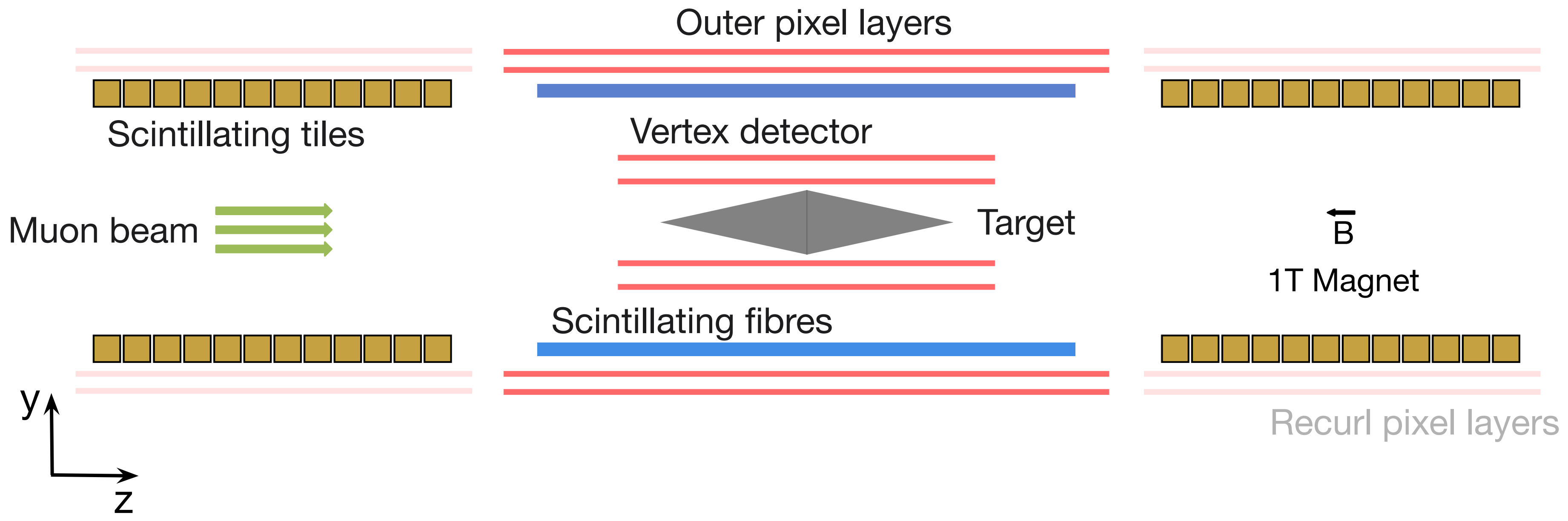
$\longrightarrow$ *Below current or foreseeable experimental sensitivity*



Muon decay via neutrino mixing

Observing this decay would be a direct evidence for physics beyond the Standard Model

Mu3e Detector: Phase 0



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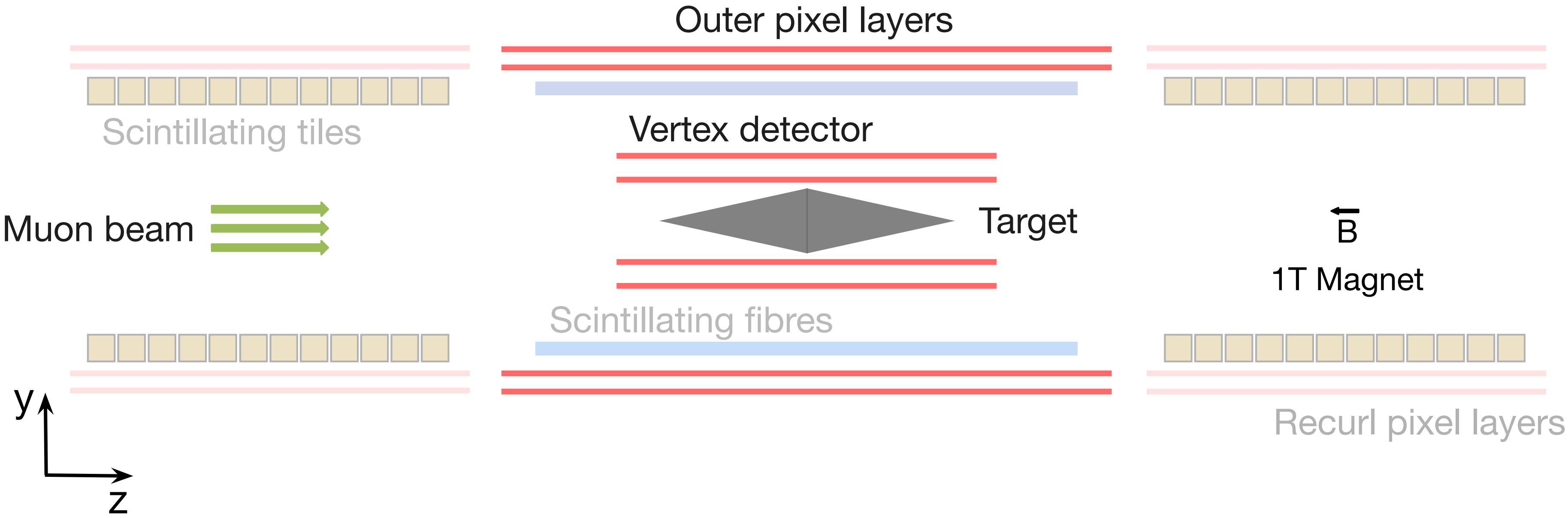

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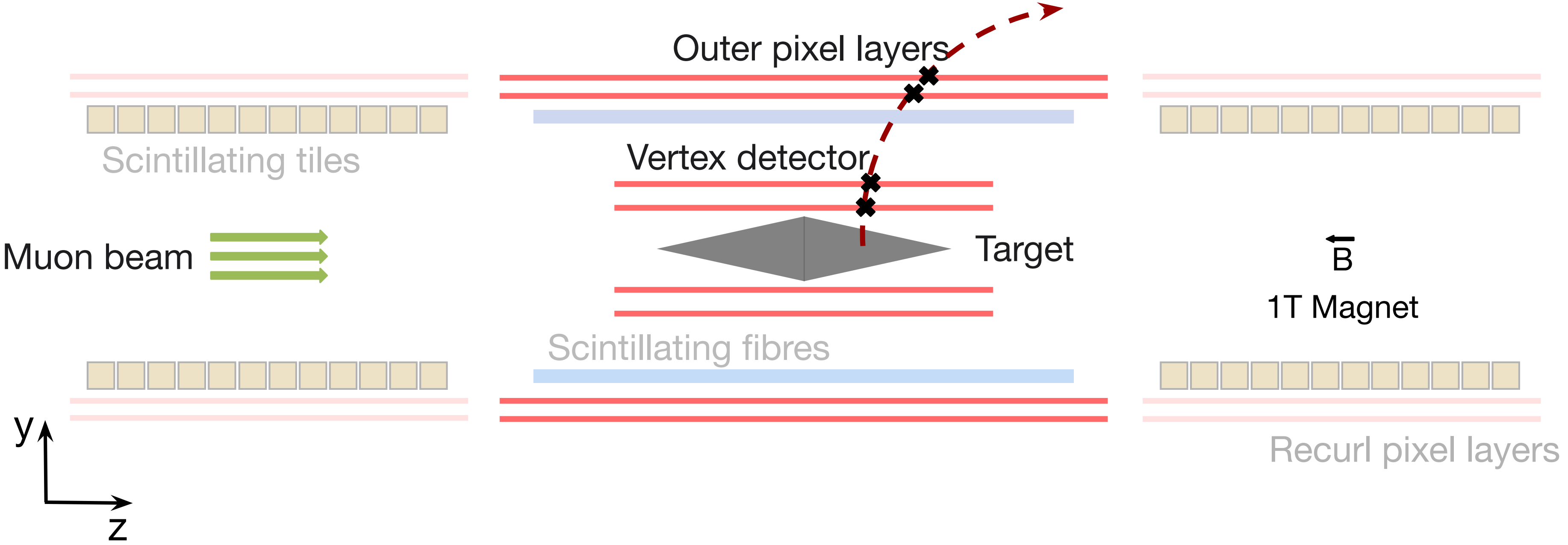

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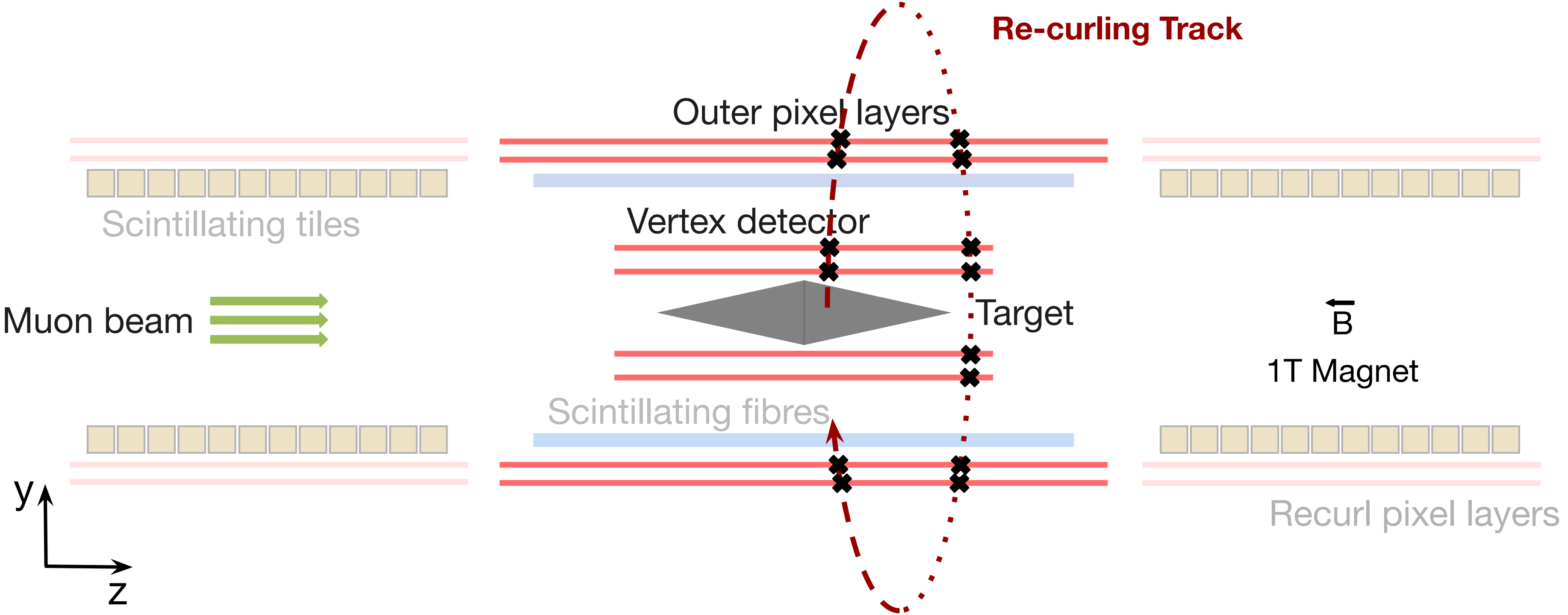
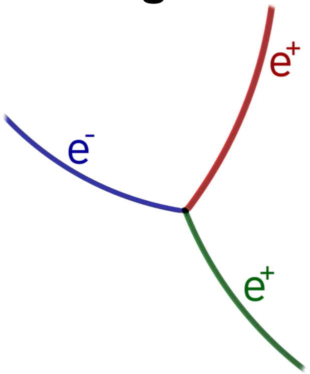


Signal and Background Topologies

Signal



Common vertex
m_{eee} = Muon mass
$\sum p_e = 0$
Coincident in time

Signal and Background Topologies

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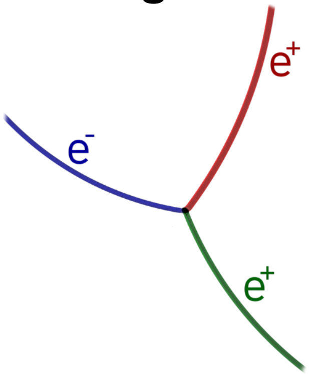
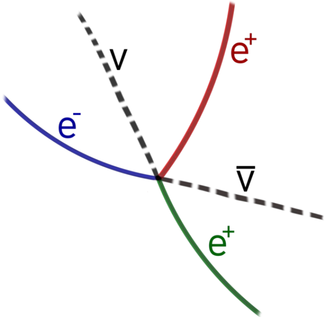


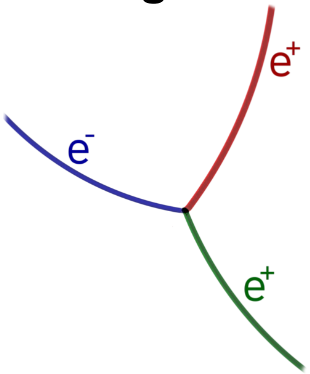
Internal Conversion



Common vertex	**Common vertex**
m_{eee} = Muon mass	m_{eee} < Muon mass
$\sum p_e = 0$	$\sum p_e > 0$
Coincident in time	Coincident in time

Signal and Background Topologies

Signal



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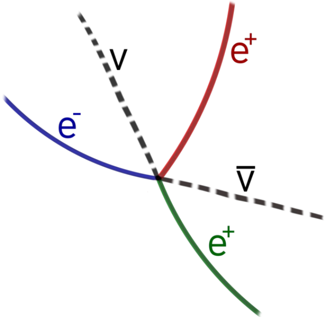
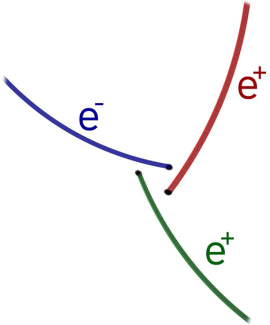


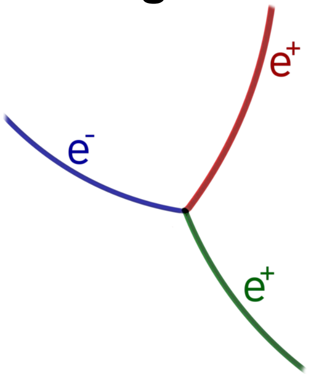
Combinatorial Background



Common vertex	**Common vertex**	**No common vertex**
m_{eee} = Muon mass	m_{eee} < Muon mass	$m_{eee} \neq$ Muon mass
$\sum p_e = 0$	$\sum p_e > 0$	$\sum p_e > 0$
Coincident in time	Coincident in time	Not coincident in time

Signal and Background Topologies

Signal



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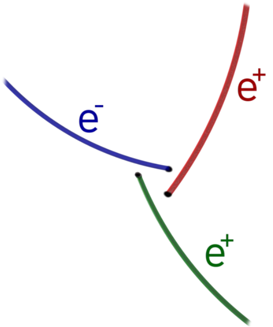

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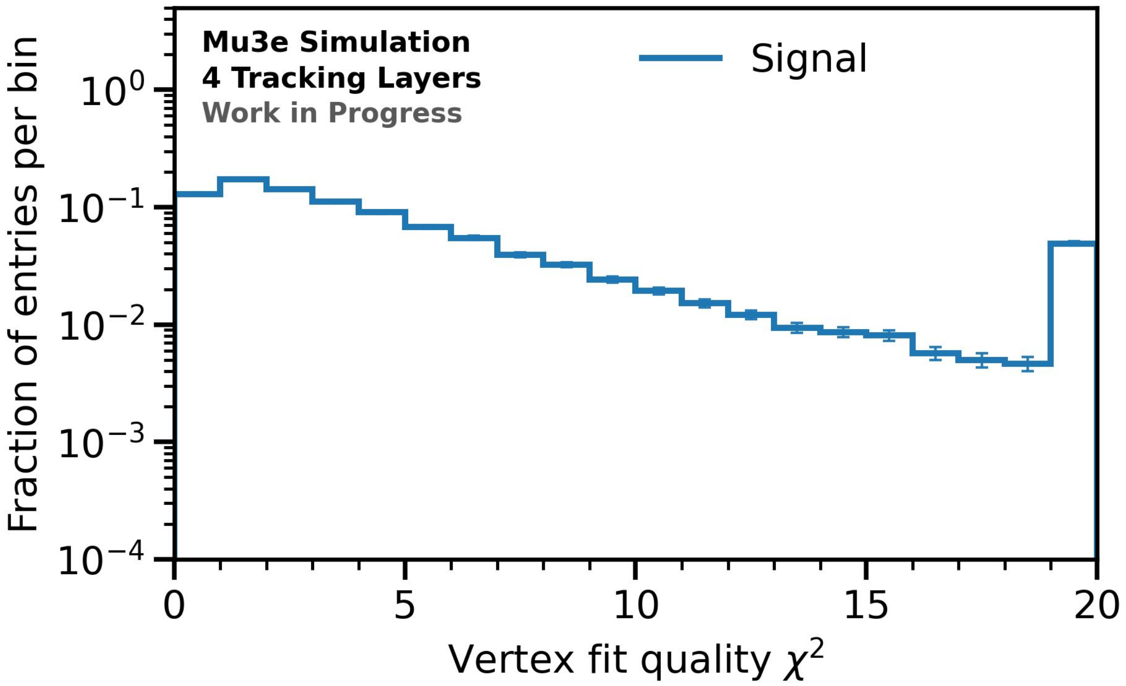
Mu3e Experiment: Results

Mu3e Geant4 Simulation of
Signal and

Events with:

- ❏ Recurling tracks
- ❏ Correctly reconstructed tracks (Hits belong to the same MC particle)



Mu3e Experiment: Results

Mu3e Geant4 Simulation of **Signal** and **Combinatorial Background**

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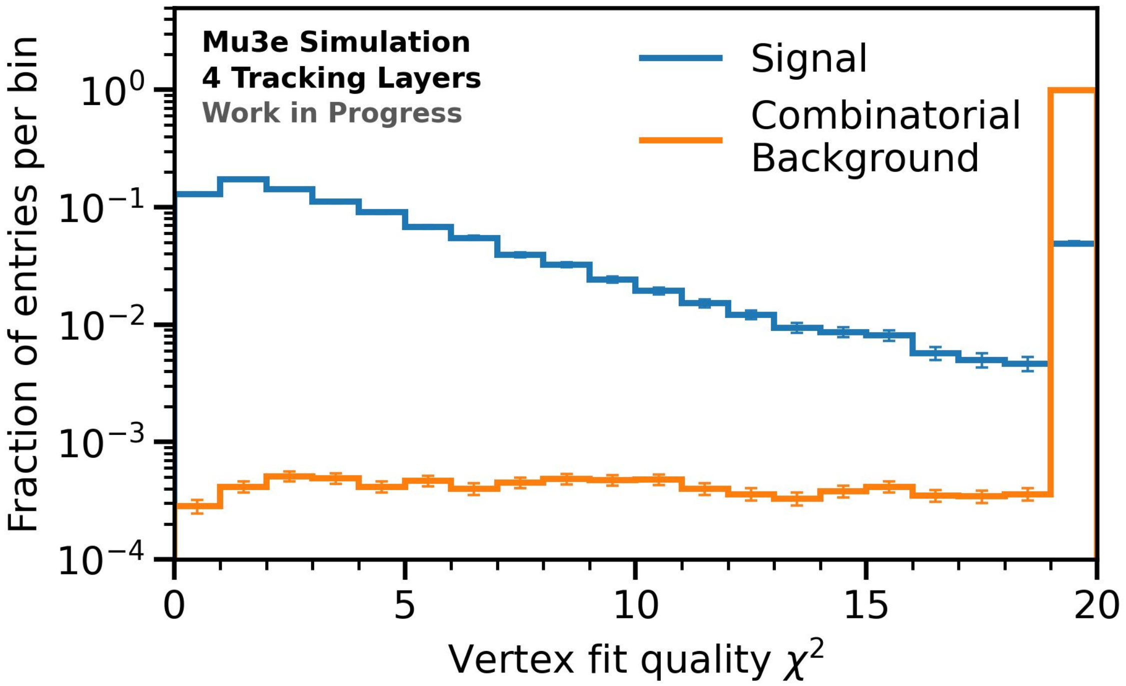
- ❏ Recurling tracks
- ❏ Correctly reconstructed tracks (Hits belong to the same MC particle)



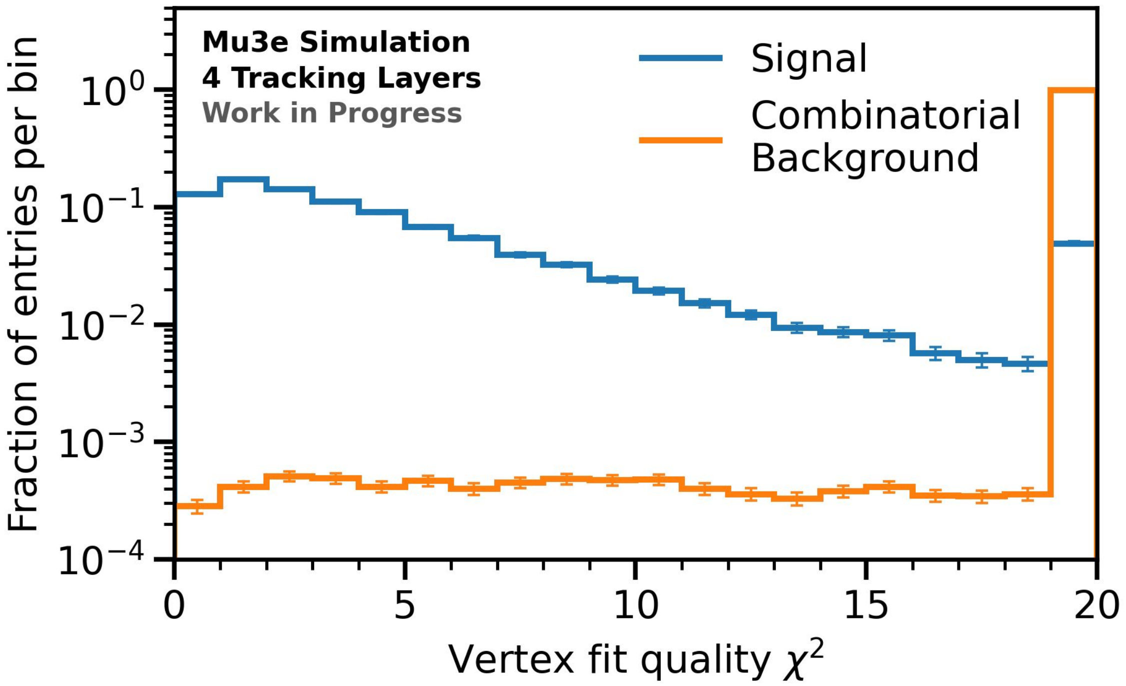
Mu3e Experiment: Results

Mu3e Geant4 Simulation of **Signal** and **Combinatorial Background**

Events with:

- ❏ Recurling tracks
- ❏ Correctly reconstructed tracks (Hits belong to the same MC particle)

The GTTF Vertex Fit quality provides a strong discrimination power between signal and background



Summary

Extension of the GTTF to include vertex reconstruction

Validation of the Vertex Fit with a Toy Detector

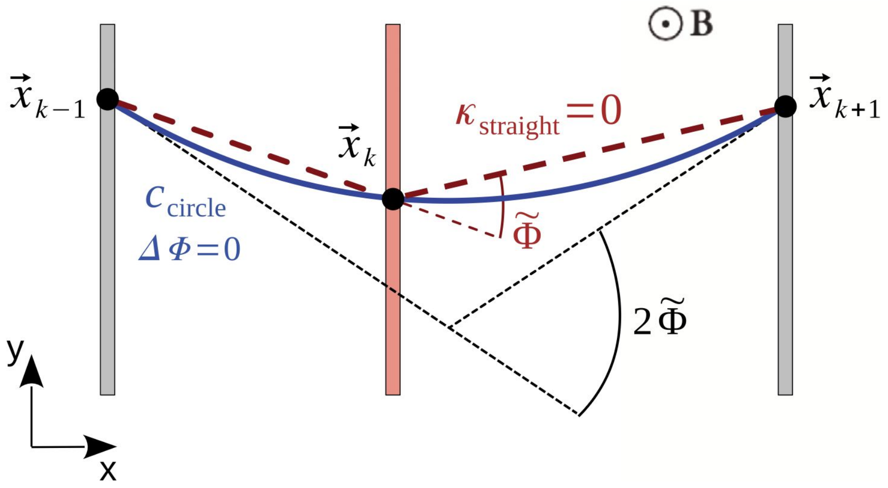
Suitable for Use in the Mu3e Experiment

Backup

General Triplet Track Fit: Linearization

Kink angle at layer *k* can be linearized around a reference solution.

$$\Delta\phi = 0 + (\kappa - \kappa_{\text{ref}})\,\rho_\phi + O(\kappa^2) \approx \tilde{\Phi} + \rho_\phi\,\kappa,$$
$$\Delta\theta = \Delta\theta_{\text{ref}} + (\kappa - \kappa_{\text{ref}})\,\rho_\theta + O(\kappa^2) \approx \tilde{\Theta} + \rho_\theta\,\kappa\,.$$



Four parameters describe the curvature dependence of the kink angles.

Sketch of a hit triplet with zero kink angle solution and zero curvature solution

General Triplet Track Fit: Linearization

The χ^2 function can be re-written with the linearization:

Fit parameter:

- 3D curvature κ
- Hit position shifts δ

→ Minimizing this χ^2 function gives a system of linear equations that can be solved!

MS uncertainties

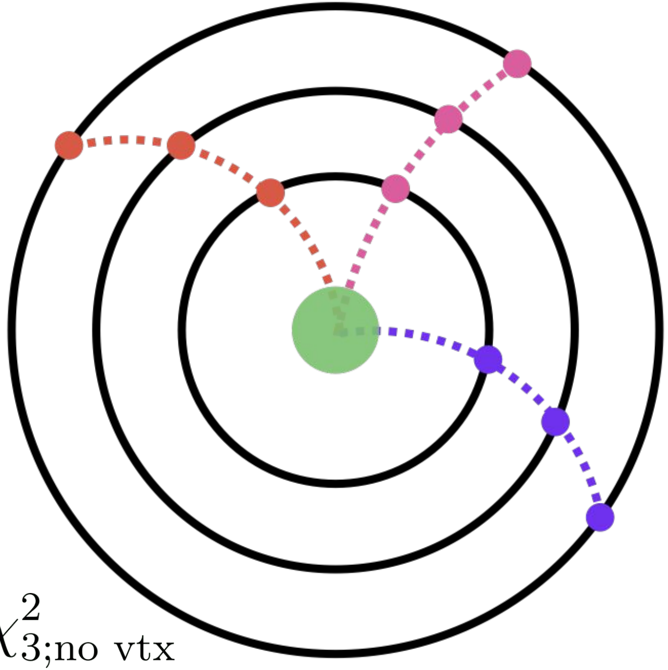
$$\chi_i^2(\kappa_i, \vec{\boldsymbol{\delta}}_{\boldsymbol{i}}) = \boxed{\sum_{j=0}^{n_{hit\,i}-2} \frac{\left(\tilde{\Theta}_{ij} + \rho_{\theta_{ij}} \kappa_{ij} - \Delta\Theta_{\text{hit},ij}(\vec{\boldsymbol{\delta}}_{\boldsymbol{i}})\right)^2}{\sigma^2_{\Theta_{\text{MS}},ij}} + \sum_{j=0}^{n_{hit\,i}-2} \frac{\left(\tilde{\Phi}_{ij} + \rho_{\phi_{ij}} \kappa_{ij} - \Delta\Phi_{\text{hit},ij}(\vec{\boldsymbol{\delta}}_{\boldsymbol{i}})\right)^2}{\sigma^2_{\Phi_{\text{MS}},ij}}} + \boxed{\sum_{k=1}^{n_{hit\,i}} \vec{\delta}_{ik}^{\,T}\, \vec{\vec{V}}{}'^{-1}_{ik}\, \vec{\delta}_{ik}}$$

hit uncertainties

General Triplet Track Fit: Vertex Fit

- Vertex Fit:

$$\chi^2_{\mathrm{GTTVertex}} = \chi^2_1 + \chi^2_2 + \chi^2_3 + \vec{\delta}^{\,T}_{\mathrm{vtx}} \mathbf{V}_{\mathrm{vtx}} \vec{\delta}_{\mathrm{vtx}}$$



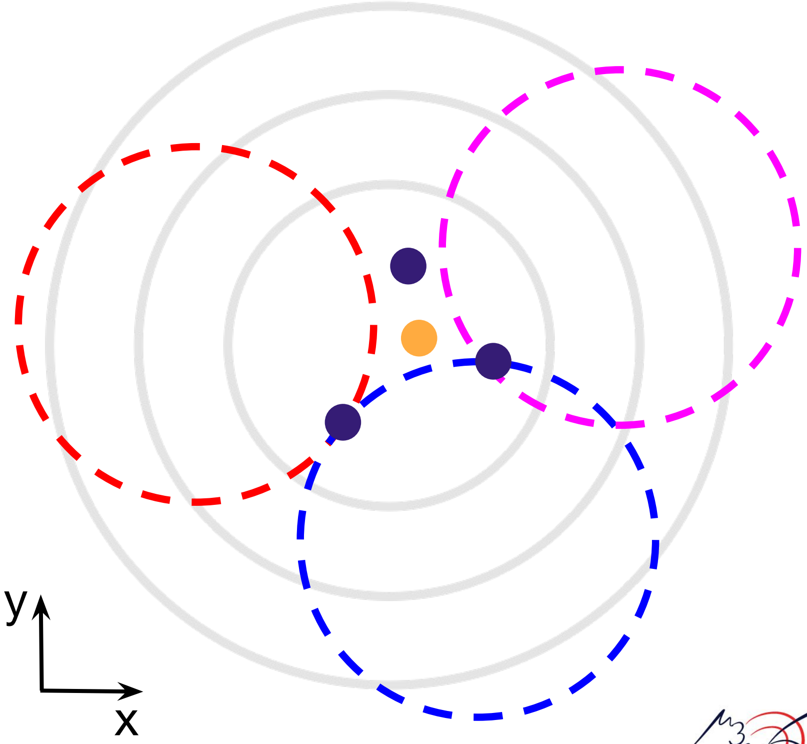
- Pure Vertex Chi2:
  - GTTVertex Fit fits vertex + tracks at the same time
  - Vertex Chi2 should only describe the quality of the vertex position fit

$$\chi^2_{\mathrm{Vertex}} = \chi^2_{\mathrm{GTTVertex}} - \chi^2_{1;\mathrm{no\ vtx}} - \chi^2_{1;\mathrm{no\ vtx}} - \chi^2_{3;\mathrm{no\ vtx}}$$

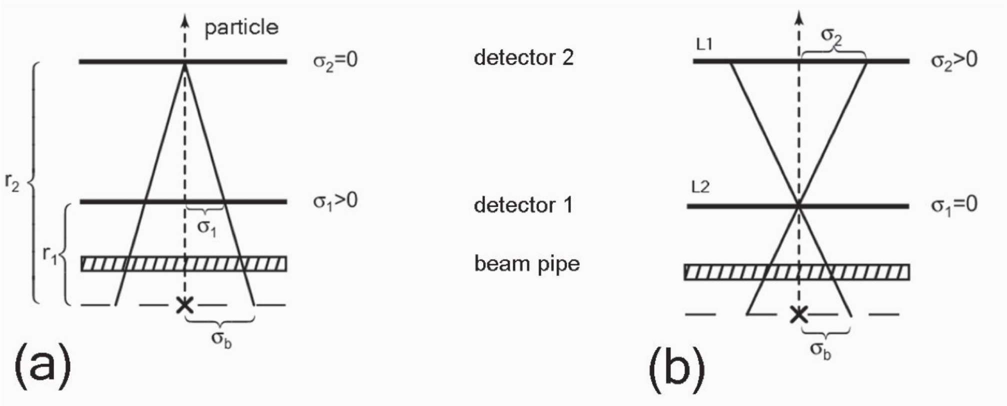
General Triplet Track Fit: Vertex Fit

Initial Vertex Guess

- Find intersection points for all tracks in xy plane
- Evaluate the z-coordinates
- Average the intersection points



Toy Detector: Two Layer Model



Assuming one of the layers has a perfect resolution, it follows rom geometrical arguments:

$$\frac{\sigma_{\mathrm{vtx}}}{\sigma_2} = \frac{r_1}{r_2 - r_1}$$

$$\frac{\sigma_{\mathrm{vtx}}}{\sigma_1} = \frac{r_2}{r_2 - r_1}$$

$$\sigma_{\mathrm{vtx}} = \sqrt{\left(\frac{r_1}{r_2 - r_1}\sigma_2\right)^2 + \left(\frac{r_2}{r_2 - r_1}\sigma_1\right)^2}$$

Toy Detector: Two Layer Model

Final Equation: Expected Vertex Resolution

$$\sigma_{\mathrm{vtx}} = \sqrt{\frac{(r_1^2 + r_2^2)}{(r_2 - r_1)^2} \cdot \frac{d^2}{12} + r_1^2 \left(\frac{13.6\,\mathrm{MeV/c}}{p \cdot \beta} \sqrt{\frac{x}{X_0}} \left(1 + 0.038 \cdot \ln \left(\frac{x}{X_0} \right) \right) \right)^2}$$