D5-Branes and Quantum Impurity Models

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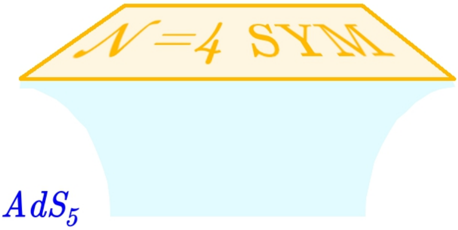
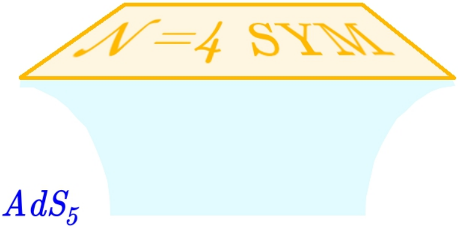
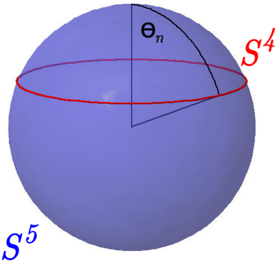
W.M. arXiv:1012.1973
A. Faraggi, W.M., L. A. Pando Zayas arXiv:1112.5028



Classical SUGRA regime

$$N \to \infty, \qquad \text{large } \lambda$$

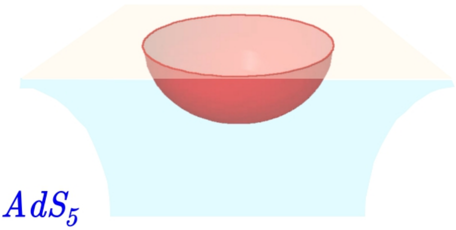


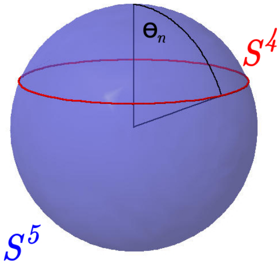


Latitude angle θ_n is quantized by fundamental string charge

$$n = \frac{N}{\pi}(\theta_n - \sin\theta_n \cos\theta_n) \qquad 0 \le n \le N\,, \quad \frac{n}{N} \text{ fixed}$$

[Pawelczyk, Rey hep-th/0007154; Camino, Paredes, Ramallo hep-th/0104082]





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$$n = \frac{N}{\pi}(\theta_n - \sin\theta_n \cos\theta_n) \qquad 0 \le n \le N\,, \quad \frac{n}{N} \text{ fixed}$$

2-d part of the worldvolume behaves like a string worldsheet

effective string tension $\quad T_{eff} = \dfrac{N}{3\pi^2\alpha'}\sin^3\theta_n$

[Hartnoll hep-th/0606178]

D5-branes $\leftrightarrow$ $SU(N)$ antisymmetric operators in $\mathcal{N} = 4$ SYM

$$\text{D5-brane with fundamental string charge } n \quad \longleftrightarrow \quad \Gamma_n = \left.\begin{array}{c}\square\\ \vdots\\ \square\end{array}\right\} n$$

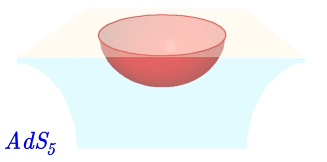
D5-branes $\leftrightarrow$ $SU(N)$ antisymmetric operators in $\mathcal{N} = 4$ SYM

$$\text{D5-brane with fundamental string charge } n \quad \longleftrightarrow \quad \Gamma_n = \left.\begin{matrix} \square \\ \vdots \\ \square \end{matrix}\right\} n$$

Example: Circular Wilson loop of antisymmetric operators

D5-brane renormalized on-shell action

$$I_{\text{on-shell}} = -\frac{2N\sqrt{\lambda}}{3\pi} \sin^3 \theta_n$$



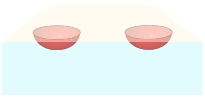
agrees with matrix model

$$e^{-I_{\text{on-shell}}} = \left\langle \text{Tr}_{\Gamma_n}[e^M] \right\rangle \equiv \frac{1}{Z} \int dM \, \text{Tr}_{\Gamma_n}[e^M] \exp\left(-\frac{2N}{\lambda} \text{Tr}[M^2] \right)$$

[Yamaguchi hep-th/0603208; Hartnoll, Kumar hep-th/0605027]

Motivation

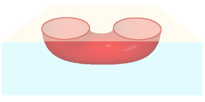
Two Wilson loops



action is independent of distance between loops

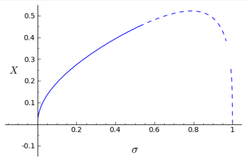
$$I = 2I_{WL} = -\frac{4N\sqrt{\lambda}}{3\pi}\sin^3\theta_n$$

Wilson loop correlator

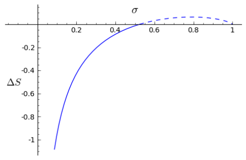


shape depends on order parameter σ

distance between loops/radius

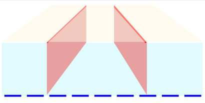


on-shell action difference



[Zarembo hep-th/9904149] for fundamental representation

Two Polyakov loops

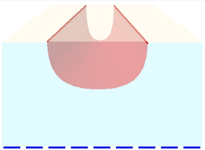


finite temperature, D5-branes end at horizon
action is independent of distance between loops

$$I = I_{PL} = -2\frac{N\sqrt{\lambda}}{3\pi}\sin^3\theta_n$$

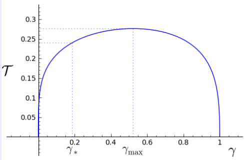
note: $I_{WL} = 2I_{PL}$

Holographic dimer

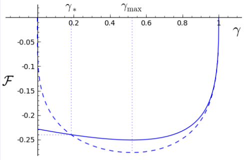


shape depends on order parameter γ

"temperature" $T = \frac{X}{\beta}$



free energy

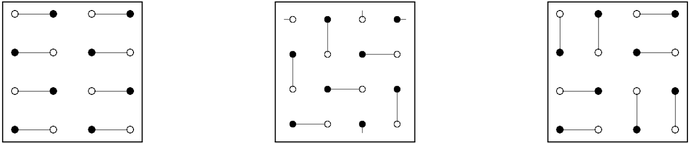


[Kachru, Karch, Yaida arXiv:0909.2639], older work on $q-\bar{q}$ potential

Spin glasses

holographic dimer → spin glasses, Fermi/non-Fermi liquid phase transition

[Kachru, Karch, Yaida arXiv:0909.2639, 1009.3268]



Quantum impurity model

$$I = I_{\mathcal{N}=4} + \int dt\,[i\chi^\dagger \partial_t \chi + \chi^\dagger (A_0 + \phi)\chi + \mu(\chi^\dagger \chi - n)]$$

[Gomis, Passerini hep-th/0604007]

This is a Kondo-like quantum impurity model similar to those describing quantum antiferromagnets. ⇒ Physics of strange metals

[Sachdev arXiv:1006.3794, 1010.0682]

Questions addressed in this seminar

Part I: Quantum impurity model

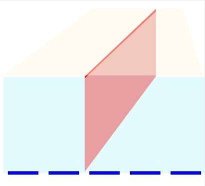
- field theoretic calculation of Wilson and Polyakov loop expectation values
  $\Rightarrow$ thermodynamic explanation of $\sqrt{\lambda}$ entropy enhancement
- understand phase transitions from a field theoretic point of view (future)

Part II: D5-brane fluctuations

- field equations for bosonic and fermionic fluctuations
- spectrum of fluctuations for circular Wilson loop
  $\Rightarrow$ confirmation of $OSp(4^*|4)$ multiplet structure
- one-loop effective action for circular Wilson loop
  $\Rightarrow$ compare with matrix model beyond leading order (future)

Part I: Quantum impurity model

D5-brane calculation of Polyakov loop on $\mathbb{R}^3 \times \mathbb{R}$



- finite temperature (black brane gravity background)

$$\beta = \frac{\pi l^2}{r_+}$$

- renormalized on-shell action

$$I = -\beta F = -\frac{N\sqrt{\lambda}}{3\pi^2 l^2} \beta r_+ \sin^3 \theta_n$$

- strong coupling entropy enhancement

$$S = \frac{N\sqrt{\lambda}}{3\pi} \sin^3 \theta_n \qquad \nu = \frac{n}{N} = \frac{1}{\pi}(\theta_n - \sin\theta_n \cos\theta_n)$$

- notice similarity with circular Wilson loop, $S = \frac{1}{2} \ln \langle W \rangle$
- in contrast, weak coupling degeneracy of states is

$$\ln d_n = -N[\nu \ln \nu + (1-\nu)\ln(1-\nu)]$$

$\mathcal{N} = 4$ SYM with fermionic impurity

$$I = \int d^3x\, d\tau\, \mathcal{L}_{\mathcal{N}=4} + \int d\tau \left[\chi_a^\dagger \partial_\tau \chi^a + i\chi_a^\dagger \tilde{A}_b^a \chi^b + i\mu(\chi_a^\dagger \chi^a - n) \right]$$

- χ : N-component spinor
- $\tilde{A}$: $\tilde{A} = A_\tau + v^I \phi_I$, $\tilde{A}_b^a = (t^c)^a{}_b \tilde{A}_c$
- μ : Lagrange multiplier, fixes fermion occupation number to n
- τ : Euclidean time, $\tau \in (0, \beta)$

$\mathcal{N}=4$ SYM with fermionic impurity

$$I=\int d^3x\,d\tau\,\mathcal{L}_{\mathcal{N}=4}+\int d\tau\left[\chi_a^\dagger\partial_\tau\chi^a+i\chi_a^\dagger\tilde{A}_b^a\chi^b+i\mu(\chi_a^\dagger\chi^a-n)\right]$$

- χ : N-component spinor
- $\tilde{A}$: $\tilde{A}=A_\tau+v^I\phi_I,\ \tilde{A}_b^a=(t^c)^a_{\ b}\tilde{A}_c$
- μ : Lagrange multiplier, fixes fermion occupation number to n
- τ : Euclidean time, $\tau\in(0,\beta)$

Integrate out SYM fields $\longrightarrow$ impurity action

$$I=\int d\tau[\chi_a^\dagger(\partial_\tau+i\mu)\chi^a-i\mu n]$$
$$+\frac{\lambda}{2N}\int d\tau d\tau' D(\tau-\tau')\chi_a^\dagger(\tau)\chi_b^\dagger(\tau')\chi^b(\tau)\chi^a(\tau')$$

with SYM background correlator $\langle\tilde{A}_c(\tau)\tilde{A}_{c'}(\tau')\rangle=\frac{2\lambda}{N}\delta_{cc'}D(\tau-\tau')$

Large-N limit saddle point

2-point Green's function

$$\left\langle \mathscr{T} \chi^a(\tau) \chi_b^\dagger(0) \right\rangle = -G(\tau)\delta_b^a$$

Fourier transform

$$G(i\omega_n) = \int_0^\beta d\tau \, \mathrm{e}^{i\omega_n \tau} G(\tau)$$

analytic continuation $i\omega_n \to \omega + i0^+$ gives *retarded* Green's function

Saddle point equations

$$G(i\omega_n) = \frac{1}{i\omega_n - \bar{\mu} - \Sigma(i\omega_n)}$$ Dyson's equation ($\bar{\mu} = i\mu$)

$$\Sigma(\tau) = \lambda D(\tau) G(\tau)$$ self-energy

$$G(\tau \to 0^-) = \frac{n}{N} = \nu$$ occupation number constraint

Scaling ansatz

$$D(\tau) = A_0 \beta^{-2\Delta_0} \left[\frac{\pi}{\sin(\pi\tilde{\tau})} \right]^{2\Delta_0} \qquad G(\tau) = -A\beta^{-2\Delta} e^{\alpha\tilde{\tau}} \left[\frac{\pi}{\sin(\pi\tilde{\tau})} \right]^{2\Delta}$$

$\tilde{\tau}$: $\tilde{\tau} = \tau/\beta \in (0,1)$

Δ, Δ_0 : scaling dimensions

A, A_0 : constants

α : particle-hole asymmetry parameter

This is a generalization of the $\nu = \frac{1}{2}$ ($\alpha = 0$) case solved by Sachdev [arXiv:1010.0682].

Properties of the solution

$$2\Delta + \Delta_0 = 1 \qquad \nu = \frac{1}{2} - \frac{\vartheta}{\pi} - \frac{\Delta_0}{2\sin(\pi\Delta_0)} \sin(2\vartheta)$$

ϑ : spectral asymmetry angle $\quad e^{\alpha} = \frac{\sin(\pi\Delta - \vartheta)}{\sin(\pi\Delta + \vartheta)} \quad \vartheta \in (-\pi\Delta, \pi\Delta)$

Impurity model

$$\vartheta \in (-\pi\Delta, \pi\Delta)$$

$$\nu = \frac{1}{2} - \frac{\vartheta}{\pi} - \frac{\Delta_0}{2\sin(\pi\Delta_0)} \sin(2\vartheta)$$

D5-brane

$$\theta_n \in (0, \pi)$$

$$\nu = \frac{1}{\pi}\left(\theta_n - \sin\theta_n \cos\theta_n\right)$$

Impurity model

$$\vartheta \in (-\pi\Delta, \pi\Delta)$$

$$\nu = \frac{1}{2} - \frac{\vartheta}{\pi} - \frac{\Delta_0}{2\sin(\pi\Delta_0)}\sin(2\vartheta)$$

D5-brane

$$\theta_n \in (0, \pi)$$

$$\nu = \frac{1}{\pi}(\theta_n - \sin\theta_n \cos\theta_n)$$

Relation between impurity model and D5-brane

The D5-brane corresponds to the special case

$$\Delta_0 = 0 \qquad \Delta = \frac{1}{2} \qquad \vartheta = \frac{\pi}{2} - \theta_n$$

Background Green's funtion

$$D(\tau) = \frac{c^2}{\beta^2} \qquad \text{for dimensional reasons}$$

Self-energy equation is now trivial

$$\Sigma(\tau) = \lambda D(\tau) G(\tau) = \hat{\lambda} G(\tau) \qquad \hat{\lambda} = \frac{c^2 \lambda}{\beta^2}$$

Rescale to remove explicit β and λ from saddle point equations

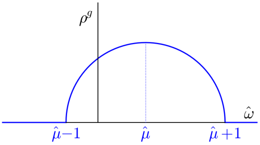
$$\omega = 2\sqrt{\hat{\lambda}}\,\hat{\omega} \qquad \bar{\mu} = 2\sqrt{\hat{\lambda}}\,\hat{\mu} \qquad G(\omega) = \frac{g(\hat{\omega})}{2\sqrt{\hat{\lambda}}}$$

Solution for retarded Green's function

$$g(\hat{\omega}) = 2\left\{(\hat{\omega} - \hat{\mu}) - \left[(\hat{\omega} - \hat{\mu})^2 - 1\right]^{1/2}\right\}$$

Spectral density is Wigner semi-circle

$$\rho^g(\hat\omega) = \begin{cases} 4\sqrt{1-(\hat\omega-\hat\mu)^2} & \text{for } |\hat\omega-\hat\mu|<1, \\ 0 & \text{otherwise.} \end{cases}$$



Occupation number

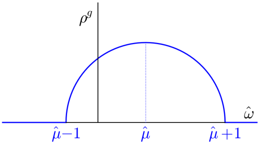
$$\nu - \frac{1}{2} = -\frac{1}{4\pi}\int dx\,\rho^g(x)\tanh\left(c\sqrt{\lambda}x\right)$$

for large λ (SUGRA regime)

$$\nu - \frac{1}{2} = -\frac{1}{\pi}\left(\arcsin\hat\mu + \hat\mu\sqrt{1-\hat\mu^2}\right) \qquad \hat\mu \in (-1,1)$$

Spectral density is Wigner semi-circle

$$\rho^g(\hat{\omega}) = \begin{cases} 4\sqrt{1-(\hat{\omega}-\hat{\mu})^2} & \text{for } |\hat{\omega}-\hat{\mu}| < 1, \\ 0 & \text{otherwise.} \end{cases}$$



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for large λ (SUGRA regime)

$$\nu - \frac{1}{2} = -\frac{1}{\pi}\left(\arcsin\hat{\mu} + \hat{\mu}\sqrt{1-\hat{\mu}^2}\right) \qquad \hat{\mu} \in (-1,1)$$

Comparison with D5-brane

$$\hat{\mu} = \cos\theta_n \qquad \Rightarrow \qquad \nu = \frac{1}{\pi}\left(\theta_n - \sin\theta_n\cos\theta_n\right)$$

Impurity entropy

$$\frac{\partial S}{\partial v} = -N \frac{\partial \bar{\mu}}{\partial T}$$

yields

$$S = c\,\frac{4N\sqrt{\lambda}}{3\pi}\left(1 - \hat{\mu}^2\right)^{3/2} = c\,\frac{4N\sqrt{\lambda}}{3\pi}\sin^3\theta_n$$

$\sqrt{\lambda}$ entropy enhancement comes from temperature dependence of $\bar{\mu}$

Comparison with D5-brane for Polyakov loop

$$S_{\mathrm{PL}} = \frac{N\sqrt{\lambda}}{3\pi}\sin^3\theta_n \qquad \Rightarrow \qquad c_{\mathrm{PL}} = \frac{1}{4}$$

Impurity entropy

$$\frac{\partial S}{\partial \nu} = -N \frac{\partial \bar{\mu}}{\partial T}$$

yields

$$S = c\frac{4N\sqrt{\lambda}}{3\pi}\left(1-\hat{\mu}^2\right)^{3/2} = c\frac{4N\sqrt{\lambda}}{3\pi}\sin^3\theta_n$$

$\sqrt{\lambda}$ entropy enhancement comes from temperature dependence of $\bar{\mu}$

Comparison with D5-brane for Polyakov loop

$$S_{\rm PL} = \frac{N\sqrt{\lambda}}{3\pi}\sin^3\theta_n \qquad \Rightarrow \qquad c_{\rm PL} = \frac{1}{4}$$

Comparison with D5-brane for Wilson loop

we know $\langle \tilde{A}(\tau)\tilde{A}(0)\rangle_{\rm WL}$ $\quad \Rightarrow \quad$ $c_{\rm WL} = \frac{1}{2}$

perfect agreement with $\quad I_{\rm WL} = -\frac{2N\sqrt{\lambda}}{3\pi}\sin^3\theta_n$

Part II: D5-brane fluctuations

Bulk background geometry

- black 3-brane

$$ds^2 = \underbrace{-f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2}dx_i^2}_{AdS_5,\ \text{finite temp.}} + \underbrace{L^2(d\theta^2 + \sin^2\theta\, d\Omega_4^2)}_{S^5}$$

- self-dual $F_5 = dC_4$ $\qquad L^4 = 4\pi g_s N \alpha'^2$

Background D5-brane

- geometry

$$\mathcal{M}_2 \times S^4 \qquad \text{with} \qquad \mathcal{M}_2 \subset AdS_5\,, \qquad S^4 \subset S^5 \quad (\text{radius } L\sin\theta_n)$$

- $\mathcal{M}_2$ is a minimal surface
- gauge field $\mathcal{F}_{\alpha\beta} = \cos\theta_n\, g_{\alpha\beta}$ $\qquad$ (on $\mathcal{M}_2$)

Bosonic action

$$I_{D5}^{(B)} = -T_5 \int d^6\xi \sqrt{-\det(g+\mathscr{F})_{ab}} + T_5 \int C_{(4)} \wedge \mathscr{F}$$

Fluctuation fields

- gauge field (still to be gauge fixed)

$$\mathscr{A}_b \to \mathscr{A}_b + a_b$$

- normal coordinate fluctuations (fixes world-volume diffeomorphisms)

$$y^m = N^m_{\underline{i}} \chi^{\underline{i}} \quad \Rightarrow \begin{cases} SO(3) \text{ triplet } \chi^{i=\underline{2,3,4}} & \perp \mathscr{M}_2 \subset AdS_5 \\ \text{singlet } \chi^{i=\underline{5}} & \perp S^4 \subset S^5 \end{cases}$$

Fermionic action

$$I_{D5}^{(F)} = \frac{T_5}{2} \int d^6\xi \sqrt{-\det M_{ab}}\, \bar{\theta} \left(1 - \Gamma_{D5}\right) \left[(\tilde{M}^{-1})^{ab} \Gamma_b D_a \right] \theta$$

- θ: doublet of 32-component left-handed Majorana-Weyl spinors
- κ-symmetric
- $M_{ab} = (g + \mathscr{F})_{ab}$, $\tilde{M}_{ab} = g_{ab} + \sigma_3 \Gamma_{(10)} \mathscr{F}_{ab}$
- D_a contains couplings to RR-fields

[Martucci, Rosseel, Van den Bleeken, Van Proeyen hep-th/0504041]

- fix κ-symmetry and Weyl condition $\Rightarrow$ doublet of 6-d Dirac spinors
- 10-d Majorana condition $\Rightarrow$ 6-d symplectic Majorana condition

Skip details

6-d action for fermions

$$I_{D5}^{(F)} = \frac{T_5}{2} \sin^4 \theta_n \int d^6\xi \sqrt{-\det \hat{g}_{ab}}\, \bar{\theta} \left[\hat{\Gamma}^a \nabla_a + \frac{1}{4} \hat{\Gamma}^\alpha A_{\underline{ij}\alpha} \tau^{\underline{ij}} - \frac{i}{L} \hat{\Gamma}^{\underline{01}} \right] \theta$$

- $A_{\underline{ij}\alpha}$ gauge field in $SO(3)$ normal bundle
- $\tau^{\underline{ij}}$ Pauli matrices, act on doublet
- "mass term" on $\mathcal{M}_2$

6-d action for fermions

$$I_{D5}^{(F)} = \frac{T_5}{2} \sin^4 \theta_n \int d^6\xi \sqrt{-\det \hat{g}_{ab}}\, \bar{\theta} \left[\hat{\Gamma}^a \nabla_a + \frac{1}{4} \hat{\Gamma}^\alpha A_{\underline{ij}\alpha} \tau^{\underline{ij}} - \frac{i}{L} \hat{\Gamma}^{\underline{01}} \right] \theta$$

- $A_{\underline{ij}\alpha}$ gauge field in $SO(3)$ normal bundle
- $\tau^{\underline{ij}}$ Pauli matrices, act on doublet
- "mass term" on $\mathscr{M}_2$

Alternative form after a chiral rotation

$$I_{D5}^{(F)} = \frac{T_5}{2} \sin^4 \theta_n \int d^6\xi \sqrt{-\det \hat{g}_{ab}}\, \bar{\theta} \left[\hat{\Gamma}^a \nabla_a + \frac{1}{4} \hat{\Gamma}^\alpha A_{\underline{ij}\alpha} \tau^{\underline{ij}} + \frac{1}{L} \hat{\Gamma}^{\underline{6789}} \right] \theta$$

- "mass term" on S^4

Skip details

Quadratic action for bosons

$$I_{D5}^{(B,2)} = T_5 \sin^3\theta_n \int d^6\xi \sqrt{-\det \hat{g}_{ab}} \Big[-\frac{1}{2}\hat{g}^{ab}\left(\delta_{\underline{ij}}\nabla_a\chi^{\underline{i}}\nabla_b\chi^{\underline{j}} + \nabla_a\chi^{\underline{5}}\nabla_b\chi^{\underline{5}}\right)$$
$$+\frac{2}{L^2}(\chi^{\underline{5}})^2 - \frac{1}{4\sin^2\theta_n}\hat{g}^{ab}\hat{g}^{cd}f_{ac}f_{bd} - \frac{2}{L\sin\theta_n}\chi^{\underline{5}}\varepsilon^{\alpha\beta}f_{\alpha\beta}$$
$$+\frac{1}{2}\left(H_{\underline{i}\alpha\beta}H_{\underline{j}}{}^{\alpha\beta} + R_{mpnq}\,g^{\alpha\beta}x^m_\alpha x^n_\beta N^p_{\underline{i}}N^q_{\underline{j}}\right)\chi^{\underline{i}}\chi^{\underline{j}}\Big]$$

- $H_{\underline{i}\alpha\beta}$: second fundamental form on $\mathscr{M}_2$
- x^m_α: tangent vectors
- $\hat{g}_{ab}$: effective 6-d metric

$$d\hat{s}^2 = g_{\alpha\beta}d\xi^\alpha d\xi^\beta + L^2 d\Omega_4^2$$

- ∇_a: $SO(3)$ covariant derivative

Gauge fix a_a

- Lorentz gauge $\hat{\nabla}_b a^b = 0$
- *on-shell*, impose transversality on $\mathscr{M}_2$ and S^4: $\hat{\nabla}_\alpha a^\alpha = \hat{\nabla}_\mu a^\mu = 0$

Decompose into spherical harmonics on S^4

$$\chi^j = \sum_{l=0}^{\infty} \chi_l^j(\tau,\rho) Y_l(\Omega) \qquad \chi^5 = \sum_{l=0}^{\infty} \chi_l^5(\tau,\rho) Y_l(\Omega)$$

$$a^\alpha = \sum_{l=0}^{\infty} a_l^\alpha(\tau,\rho) Y_l(\Omega) \qquad a^\mu = \sum_{l=0}^{\infty} a_l(\tau,\rho) Y_{l+1}^\mu(\Omega)$$

Diagonalize modes in mixed sector

$$\zeta_l = f_l + \frac{\sin\theta_n}{L}(l-1)\chi_l^5 \qquad \eta_l = f_l - \frac{\sin\theta_n}{L}(l+4)\chi_l^5$$

($f = \varepsilon^{\alpha\beta}\nabla_\alpha a_\beta$, η_0 is not dynamical)

Mode field equations on $\mathscr{M}_2$

$$\left[\left(\nabla^\alpha\nabla_\alpha - \frac{l(l+3)}{L^2}\right)\delta^{\underline{i}}_{\underline{j}} + H^{\underline{i}}{}_{\alpha\beta}H_{\underline{j}}{}^{\alpha\beta} + R_{mpnq}\,g^{\alpha\beta}x^m_\alpha x^n_\beta N^{\underline{i}p}N^q_{\underline{j}}\right]\chi^{\underline{j}}_l = 0$$

$$\left(\nabla^\alpha\nabla_\alpha - \frac{(l+2)(l+3)}{L^2}\right)a_l = 0$$

$$\left[\nabla^\alpha\nabla_\alpha - \frac{1}{L^2}(l+3)(l+4)\right]\zeta_l = 0$$

$$\left[\nabla^\alpha\nabla_\alpha - \frac{1}{L^2}l(l-1)\right]\eta_l = 0$$

- circular Wilson loop: $\mathscr{M}_2 = AdS_2 \Rightarrow$ standard AdS/CFT
- Wilson loop correlator, holographic dimer $\Rightarrow$ Heun's differential equation

6-d field equation

$$\left[\hat{\Gamma}^a \tilde{\nabla}_a - \frac{i}{L}\Gamma^{\underline{01}}\right]\theta = 0$$

Mode decomposition

$$\theta = \sum_{l,s} \chi_{ls} \otimes \psi_{ls}$$

- χ_{ls}: spinor spherical harmonics on S^4
- ψ_{ls}: doublet of Dirac spinors on $\mathscr{M}_2$

2-d field equation

$$\hat{\gamma}^\alpha \tilde{\nabla}_\alpha \psi = \pm\mu\psi \qquad \text{with } \mu = (l+1, l+3)$$

Bosons

$$\left(\nabla^\alpha \nabla_\alpha - m^2\right)\varphi = 0$$

$$h = \frac{1}{2} + \sqrt{\frac{1}{4} + m^2 L^2}$$

field	m^2L^2	$(h,n)\times(m,l)$
$\eta_l\ (l \geq 1)$	$l(l-1)$	$(l,0)\times(0,l)$
ζ_l	$(l+3)(l+4)$	$(l+4,0)\times(0,l)$
a_l	$(l+2)(l+3)$	$(l+3,0)\times(2,l)$
χ_l^i	$(l+1)(l+2)$	$(l+2,1)\times(0,l)$

Fermions

$$(\gamma^\alpha \nabla_\alpha - m)\,\psi = 0$$

$$h = |m|L + \frac{1}{2}$$

field	mL	$(h,n)\times(m,l)$
ψ_{l+}	$(l+1)$	$(l+\frac{3}{2},\frac{1}{2})\times(1,l)$
ψ_{l+}	$(l+3)$	$(l+\frac{7}{2},\frac{1}{2})\times(1,l)$

$OSp(4^*|4)$ multiplet structure

[Faraggi, Pando Zayas arXiv:1101.5145]

String $\leftrightarrow$ Wilson loop of fundamental representation

- matrix model calculation at $N = \infty$, all orders in λ

$$\langle W \rangle = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) = \mathrm{e}^{\sqrt{\lambda} - \frac{3}{4}\ln\lambda + \frac{1}{2}\ln\frac{2}{\pi} + \cdots}$$

[Erickson, Semenoff, Zarembo hep-th/0003055]

- one-loop effective action of string confirms next-to leading order, up to a numerical constant [Kruscenski, Tirziu arXiv:0803.0315]
- matrix model calculation to all orders in $1/N$, all orders in λ

[Drukker, Gross hep-th/0010274]

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D5-brane $\leftrightarrow$ Wilson loop of antisymmetric representation

D5-brane and matrix model at $N = \infty$, leading order in λ

$$\ln\langle W \rangle = \frac{2N\sqrt{\lambda}}{3\pi}\sin^3\theta_n\,, \qquad n = \frac{N}{\pi}(\theta_n - \sin\theta_n\cos\theta_n)$$

Summary of calculation

- want 1-loop effective action for fluctuations on $AdS_2 \times S^4$
- follow $AdS_2 \times S^2$ case in [Banerjee, Gupta, Mandal, Sen arXiv:1106.0080]
- use heat-kernel technique

$$\Gamma_1 = -\ln(L/L_0)\,\zeta(0) \qquad \zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty dt\, t^{s-1} Y(t)$$

- contributions from

Y^{χ^i}	triplet of scalars
Y^{a_μ}	vector and gauge fixing contribution
$Y^{\chi^5,d}$	coupled sector of singlet scalar and 2-d gauge field
$Y^{\tilde{c}}$	special 2-d gauge field modes
Y^{θ}	fermions

- result

$$\zeta(0) = -\frac{1}{12}$$

- zero modes ? ($\ln\lambda$ term in the string case)

Skip details

Peculiarities

- mass matrix in mixed sector

$$\begin{pmatrix} \nu^2+\frac{1}{4}+l(l+3)-4 & 4i\sqrt{\nu^2+\frac{1}{4}} \\ 4i\sqrt{\nu^2+\frac{1}{4}} & \nu^2+\frac{1}{4}+l(l+3) \end{pmatrix}$$

- eigenvalues

$$(\nu \pm 2i)^2+\frac{1}{4}+l(l+3)+2$$

- determinant

$$\left[\nu^2+\frac{1}{4}+l(l-1)\right]\left[\nu^2+\frac{1}{4}+(l+3)(l+4)\right]$$

- There are two ways to calculate the heat kernel. Results differ in $1/t^2$ and $1/t$ terms, but the $1/t^3$ and the constant terms are the same.
- A similar ambiguity exists for the fermions (chiral transformation).

Results

- "mass term" on AdS_2

$$Y_1(t) = \frac{V_{\widehat{AdS_2}}}{2\pi}\left(-\frac{2}{3t} + \frac{1}{12} + \cdots\right)$$

- "mass term" on S^4

$$Y_2(t) = \frac{V_{\widehat{AdS_2}}}{2\pi}\left(\frac{4}{3t} + \frac{1}{12} + \cdots\right)$$

- $1/t^3$ and $1/t^2$ terms cancel in both cases
- renormalized volume of unit AdS_2 for circular boundary

$$V_{\widehat{AdS_2}} = -2\pi$$

- final result

$$\zeta(0) = -\frac{1}{12}$$

Outlook

Quantum impurity model

- formulate and solve 2-impurity model for Wilson loop correlator and holographic dimer

Comparison of 1-loop effective action with matrix model

- sort out zero mode issues
- matrix model used by Yamaguchi and Hartnoll, Kumar is not symmetric under $n \longrightarrow N - n$ beyond leading order
- restriction to traceless matrices solves this issue, but affects the result of Drukker and Gross

Thank you!