Holographic Renormalization and Fake Supergravity

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based on work with:

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Problemi Attuali di Fisica Teorica

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Outline

Introduction

The physics of a QFT is encoded in the (formal) path integral

$$e^{-I[\mathfrak{s}]} = \int \mathcal{D}\phi \, e^{-S_{\mathrm{QFT}}[\phi] + \int dx \, \mathfrak{s}\cdot\mathcal{O}[\phi]}$$

where:

- $\mathcal{O}$: set of *all* local operators
- $\mathfrak{s}$: sources coupling to $\mathcal{O}$
- I: generating functional of connected correlation functions of $\mathcal{O}$

But if the QFT is strongly coupled, this calculation is impractical.

Generically, renormalization leads to strongly coupled regime.

Introduction

Here Comes AdS/CFT

For certain *strongly coupled* QFTs (=CFTs) (typically, large N gauge theories), there is a (classical) gravity dual

AdS/CFT Correspondence Formula

$$I[\mathfrak{s}] = S_{\text{on-shell}}[\mathfrak{s}]$$

where:

- $S_{on\text{-}shell}$: *renormalized* on-shell action of dual theory in bulk (asymptotically AdS) space-time
- $\mathfrak{s}$: suitably defined boundary values of bulk fields

The correspondence formula defines a QFT holographically

Introduction

Consistency?

QFT correlation functions must be finite and satisfy Ward identities.

Holographic Renormalization

- For aAdS bulk space-times, HR provides a systematic method to cancel divergences of bulk on-shell action by adding *local* covariant counterterms on (cut-off) boundary
- general result: finite exact one-point functions satisfying (anomalous) Ward identities

$$\langle \mathcal{O}[\mathfrak{s}] \rangle = -\frac{\delta I}{\delta \mathfrak{s}}$$

[Henningson, Skenderis; Bianchi, Freedman, Skenderis; Martelli, W.M.; Papadimitriou, Skenderis]

Holographic Renormalization Group Flows

(Fake) Supergravity (in $d+1$ Dimensions)

$$S = \int d^{d+1}x \sqrt{g} \left[-\frac{1}{4} R + \frac{1}{2} G_{ab} \partial\phi^a \partial\phi^b + V(\phi) \right]$$

with

$$V(\phi) = \frac{1}{2} G^{ab} W_a(\phi) W_b(\phi) - \frac{d}{d-1} [W(\phi)]^2$$

- "fake": [Freedman, Nuñez, Schnabl, Skenderis]
- subsectors of (gauged and ungauged) SUGRA
- (Kaluza-Klein type) compactifications of $d = 10$ SUGRA
- non-trivial compactifications (e.g., Papadopoulos-Tseytlin solution *ansatz*) [Berg, Haack, W.M.]
- describe extremal black hole attractors [Ferrara, Gnecchi, Marrani]

Holographic Renormalization Group Flows

BPS Domain Wall Solution = Gravity Dual of RG Flow

$$ds^2 = dr^2 + e^{2A(r)}\, dx_d^2 \qquad \partial_r A = -\frac{2}{d-1} W(\phi)$$

$$\phi = \phi(r) \qquad \partial_r \phi^a = G^{ab} W_b(\phi)$$

- r: dual of energy scale
- $\phi(r)$: running couplings
- holographic β-functions [de Boer, Verlinde[2]]
- holographic c-theorem [Freedman, Gubser, Pilch, Warner]
- RG flow generated either by finite source (deformed CFT) or operator vev (spontaneous breaking of conformal symmetry)

Gauge Invariant Variables

We Want to Calculate Correlators

- consider fluctuations around RG flow "vacuum"
- use results of holographic renormalization to extract QFT correlators

Problem

- metric and scalar fluctuations couple non-trivially in RG background $\Rightarrow$ e.o.m.s very complicated already at linearized level, interactions hopeless

Solution

[Bianchi, W.M., Prisco]

- infinitesimal diffeomorphisms act as gauge transformations on fluctuations (no gauge fixing of metric)
- construct gauge invariant combinations representing the physical degrees of freedom

Gauge Invariant Variables

Field Expansion

Metric: ADM formalism

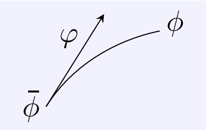
$$ds^2 = (n^2 + n_i n^i)dr^2 + 2n_i dx^i\, dr + g_{ij} dx^i dx^j$$

expansion:

$$g_{ij} = e^{2A(r)}\left(\eta_{ij} + h_{ij}\right)\ , \quad n_i = \nu_i\ , \qquad n = 1 + \nu$$

Scalars: exponential map

$$\phi^a = \bar{\phi}^a + \varphi^a - \frac{1}{2}\mathcal{G}^a_{bc}\varphi^b\varphi^c + \cdots$$



Gauge Invariant Variables

Gauge Transformations

bulk diffeomorphism

$$x^\mu \to x'^\mu = x^\mu - \xi^\mu(x)$$

acts as a gauge transformation on the fluctuations

$$\begin{aligned}
\delta\varphi^a &= W^a \xi^r + \cdots \\
\delta\nu &= \partial_r \xi^r + \cdots \\
\delta\nu^i &= \partial^i \xi^r + e^{2A}\, \partial_r \xi^i + \cdots \\
\delta h^i_j &= \partial_j \xi^i + \partial^i (\eta_{jk} \xi^k) - \frac{4}{d-1} W \delta^i_j \xi^r + \cdots
\end{aligned}$$

Gauge Invariant Variables

Gauge Invariant Field Combinations

for example,

$$\mathfrak{a}^a = \varphi^a + W^a \frac{h}{4W} + \cdots$$

$$\mathfrak{e}^i_j = h^{TT}{}^i_j + \cdots$$

$$\Rightarrow \quad \delta\mathfrak{a}^a = 0 \,, \qquad \delta\mathfrak{e}^i_j = 0$$

decomposition of h^i_j

$$h^i_j = h^{TT}{}^i_j + \partial^i \epsilon_j + \partial_j \epsilon^i + \frac{\partial^i \partial_j}{\Box} H + \frac{1}{d-1} \delta^i_j h$$

h, H and ϵ^i turn out to be the gauge degrees of freedom

Gauge Invariant Variables

Equations of Motion for Independent Degrees of Freedom

$$\left[(D_\sigma + M + d)\,(D_\sigma - M) + e^{-2\sigma}\,\frac{(d-1)^2}{4W^2}\Box\right]\mathfrak{z} = J$$

- $\sigma = A(r)$, D_σ: background covariant derivative, $\Box = \eta^{ij}\partial_i\partial_j$
- matrix M is
  - "scalars"

$$\mathfrak{z} = \mathfrak{a}^a \quad \Rightarrow \quad M^a_b = -\frac{d-1}{2} D^a D_b \ln|W|$$

  - transverse-traceless part of (d-dimensional) metric

$$\mathfrak{z} = \mathfrak{e}^i_j \quad \Rightarrow \quad M = 0$$

- J: higher order source (interaction) terms

Applications in AdS/CFT

3-Point Scattering Amplitudes *[Bianchi, W.M., Prisco]*

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle = \delta(p_1 + p_2 + p_3) \int dr \; e^{dA} \, \mathcal{X}_{123} K_1 K_2 K_3$$

$K_{1,2,3}$: bulk-to-boundary propagators (from linearized e.o.m.)

$\mathcal{X}_{123}$: operator containing $p_{1,2,3}$ and ∂_r, (from source J)

states (label n) are associated with poles in K

$$K \sim \frac{1}{p^2 + m_n^2} \tilde{K}_n$$

3-point scattering amplitude follows after amputating external legs

$$\mathcal{M}_{123} \sim \int dr \; e^{dA} \, \mathcal{X}_{123} \tilde{K}_1 \tilde{K}_2 \tilde{K}_3$$

Applications in AdS/CFT

GPPZ Flow

[Girardello, Petrini, Porrati, Zaffaroni]

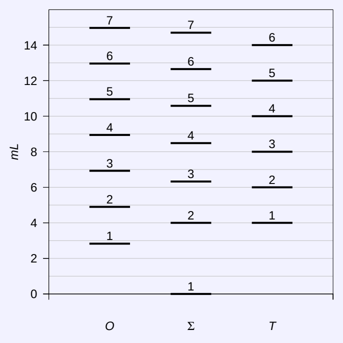
$SU(3) \times U(1)$ mass deformation of $\mathcal{N} = 4$ SYM

$$\phi \sim \mathcal{O} \in \underline{6} \text{ of } SU(3)$$
$$\sigma \sim \Sigma \in \mathrm{Tr}(W_\alpha W^\alpha)$$
$$h^i_j \sim T^i_j \in \mathrm{Tr}(W_\alpha \bar{W}_{\dot\alpha})$$

glueball spectrum

glueball	m^2
$\mathcal{O}_n$	$4n(n+1)$
Σ_n	$4(n-1)(n+2)$
T_n	$4(n+1)^2$

$(n = 1, 2, \ldots)$ [many papers]



Applications in AdS/CFT

GPPZ 3-Point Scattering Amplitudes

[Bianchi, W.M., Prisco]

Selection Rules for Decay Processes

Most kinematically allowed processes do not occur ($\mathcal{M} = 0$)

decay of Σ glueballs

$\Sigma_n \to \Sigma_m + T_{n-m-1}$ (strong)

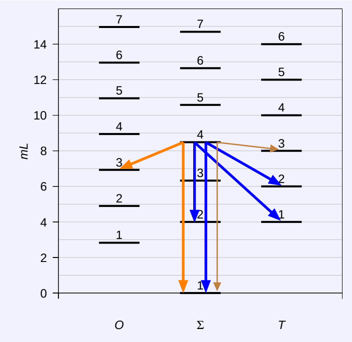
$\Sigma_n \to \Sigma_1 + \mathcal{O}_{n-1}$ (strong)

$\Sigma_n \to \Sigma_1 + T_{n-1}$ (weak)

forbidden:

$\Sigma \to \mathcal{O} + \mathcal{O}$, $\Sigma \to \mathcal{O} + T$

$\Sigma \to T + T$, $\Sigma \to \Sigma + \Sigma$



Applications in AdS/CFT

GPPZ 3-Point Scattering Amplitudes

[Bianchi, W.M., Prisco]

decay of $\mathcal{O}$ glueballs

$\mathcal{O}_n \to \Sigma_m + \Sigma_{n-m}$ (strong)

$\mathcal{O}_n \to \Sigma_1 + \Sigma_n \; (n > 1)$ (strong)

$\mathcal{O}_n \to \mathcal{O}_m + T_{n-m-1}$ (weak)

forbidden:

$\mathcal{O} \to \mathcal{O} + \mathcal{O}, \; \mathcal{O} \to T + T$

$\mathcal{O} \to \mathcal{O} + \Sigma, \; \mathcal{O} \to T + \Sigma$

$\mathcal{O}_1 \to \Sigma_1 + \Sigma_1$

$\Rightarrow$ $\mathcal{O}_1$ is stable

decay of T glueballs

$T_n \to \Sigma_m + \Sigma_{n-m}$

$T_n \to \Sigma_1 + \Sigma_n$

$T_n \to \mathcal{O}_m + \mathcal{O}_{n-m}$

forbidden:

$T \to \mathcal{O} + T$

$T \to T + T$

$T \to \mathcal{O} + \Sigma$

$T \to T + \Sigma$

T_1 is unstable

Applications in AdS/CFT

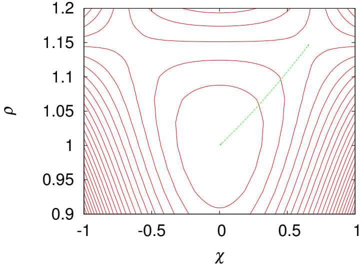
Flow to a Conformal Fixed Point in the IR

- $d = 4$: $SU(2) \times U(1)$ flow [Freedman, Gubser, Pilch, Warner]
- $d = 3$: $SU(3) \times U(1)$ flow [Ahn, Paeng, Woo]

in both:

- 2 scalar fields $\sim$ operators
- RG flow from UV to IR with operator mixing

d	Δ_{UV}	Δ_{IR}
4	2, 3	$\sqrt{7}+1, \sqrt{7}+3$
3	2, 2	$(\sqrt{17}+1)/2$ $(\sqrt{17}+5)/2$



(contour plot of W with flow solution)

Applications in AdS/CFT

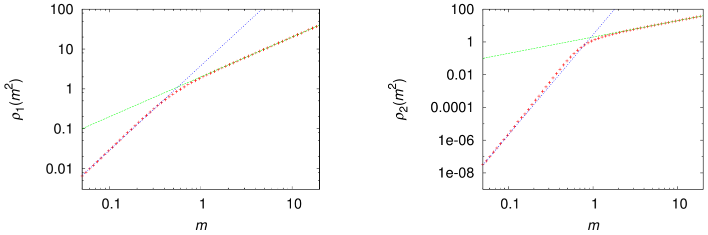
Flow to a Conformal Fixed Point in the IR

holography shows that the spectrum is continuous

Spectral Function

$$\int dx\, e^{ipx} \langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \int_0^\infty \frac{dm^2}{2\pi} \frac{\rho_{ij}(m^2)}{p^2+m^2}$$

RG Flow Exhibits Simple Cross-over from UV to IR Regime



[W.M.]

Applications in Non-AdS/Non-CFT

Important Cases with Fake SUGRA Description

- gravity duals of QFTs with logarithmic running (e.g., Klebanov-Strassler)
- Maldacena-Nuñez dual of pure $\mathcal{N} = 1$ SYM
- 0^{++} glueball sector in Klebanov-Strassler system [Berg, Haack, W.M.]
- probably also 0^{+-} and 0^{--} sectors [Benna, Dymarsky, Klebanov, Solovyov]

Pragmatic Approach to Mass Spectrum Calculation

for discrete mass spectrum impose, as in AdS/CFT

- regularity in the bulk

$$\mathfrak{a} \cdot \mathfrak{a} = G_{ab} \mathfrak{a}^a \mathfrak{a}^b < \infty$$

- integrability

$$\int dr \, e^{(d-2)A(r)} \, \mathfrak{a} \cdot \mathfrak{a} < \infty$$

Holographic Renormalization in Non-AdS/Non-CFT

Can One Do Better Than the Pragmatic Approach?

1. Can one find counterterms that yield finite correlators?
2. Is there a consistent method of holographic renormalization? Ward identities? Finite generating functional?

Partial and Encouraging Results *[Borodatchenkova, Haack, W.M.]*

1. Yes, there is a general quadratic counterterm (for 2-point functions).
2. Open question. Method is perturbative: one must go order by order.

Holographic Renormalization in Non-AdS/Non-CFT

Holographic Renormalization of 2-Point Functions

identify source (and response) functions

asymptotic behaviour of solutions (large r):

$$\mathfrak{a}^a(r) = \mathfrak{s}_\alpha\, \hat{\mathfrak{a}}^a_\alpha(r) + \mathfrak{r}_\alpha\, \check{\mathfrak{a}}^a_\alpha(r)$$

- $\hat{\mathfrak{a}}_\alpha$, $\check{\mathfrak{a}}_\alpha$: asymptotically dominant and sub-dominant solutions
- $\mathfrak{s}_\alpha$, $\mathfrak{r}_\alpha$: source coupling to $\mathcal{O}_\alpha$ and response function

define auxiliary matrices

$$\tilde{Z}_{\alpha\beta} = e^{dA}\left[(D_r\hat{\mathfrak{a}}_\alpha)\cdot\hat{\mathfrak{a}}_\beta - \hat{\mathfrak{a}}_\alpha\cdot(D_r\hat{\mathfrak{a}}_\beta)\right]$$
$$Z_{\alpha\beta} = e^{dA}\left[(D_r\hat{\mathfrak{a}}_\alpha)\cdot\check{\mathfrak{a}}_\beta - \hat{\mathfrak{a}}_\alpha\cdot(D_r\check{\mathfrak{a}}_\beta)\right]$$

field equations imply $\partial_r\tilde{Z}_{\alpha\beta} = \partial_r Z_{\alpha\beta} = 0$

Holographic Renormalization in Non-AdS/Non-CFT

Holographic Renormalization of 2-Point Functions

action encoding the linearized equations of motion

$$S_{bulk} = \frac{1}{2} \int d^d x \, dr \, e^{dA} \left[(D_r - M)\mathfrak{a} \cdot (D_r - M)\mathfrak{a} + e^{-2A} \partial_i \mathfrak{a} \cdot \partial^i \mathfrak{a} \right]$$

add local counterterm at r_{cutoff}

$$S_{ct} = \frac{1}{2} \int d^d x \, e^{dA} \, \mathfrak{a} \cdot U \cdot \mathfrak{a}$$

U: symmetric matrix, depends on r_{cutoff}, can be polynomial in $\Box$

Holographic Renormalization in Non-AdS/Non-CFT

Holographic Renormalization of 2-Point Functions

define counterterm matrix

$$U_{ab} = M_{ab} - \frac{1}{2}\left[(D_r\hat{\mathfrak{a}}_\alpha)_a(\hat{\mathfrak{a}}^{-1})_{\alpha b} + (D_r\hat{\mathfrak{a}}_\alpha)_b(\hat{\mathfrak{a}}^{-1})_{\alpha a}\right]$$

$(\hat{\mathfrak{a}}^{-1})_{\alpha a}$: matrix inverse of $\hat{\mathfrak{a}}^a_\alpha$

2-point function

$$\langle\mathcal{O}_\alpha\mathcal{O}_\beta\rangle = \lim_{r_{cutoff}\to\infty}\frac{\delta^2}{\delta\mathfrak{s}_\alpha\delta\mathfrak{s}_\beta}(S_{bulk}+S_{ct})$$

is finite:

$$\langle\mathcal{O}_\alpha\mathcal{O}_\beta\rangle = Z_{\alpha\gamma}\frac{\delta\mathfrak{r}_\gamma}{\delta\mathfrak{s}_\beta} + \frac{1}{2}\tilde{Z}_{\alpha\beta}$$

This agrees with known result in AdS/CFT case

Holographic Renormalization in Non-AdS/Non-CFT

Scheme Dependence

choice of asymptotic basis is not unique

redefinition

$$\hat{\mathfrak{a}}'_\alpha = \Lambda_{\alpha\beta}\, \hat{\mathfrak{a}}_\beta + \lambda_{\alpha\beta}\, \check{\mathfrak{a}}_\beta$$
$$\check{\mathfrak{a}}'_\alpha = \mu_{\alpha\beta}\, \check{\mathfrak{a}}_\beta$$

changes 2-point function

$$\left\langle \mathcal{O}'_\alpha \mathcal{O}'_\beta \right\rangle = \Lambda_{\alpha\gamma}\Lambda_{\beta\delta} \left\langle \mathcal{O}_\gamma \mathcal{O}_\delta \right\rangle - \frac{1}{2}(\Lambda_{\alpha\gamma}\lambda_{\beta\delta} + \Lambda_{\beta\gamma}\lambda_{\alpha\delta}) Z_{\gamma\delta}$$

$\Lambda_{\alpha\beta}$: rotates operator basis

$\lambda_{\alpha\beta}$: changes renormalization scheme
There are schemes with $Z_{\alpha\beta} = \delta_{\alpha\beta}$ and $\tilde{Z}_{\alpha\beta} = 0$.

Work in Progress

Holographic Renormalization with Gauge Invariant Variables

- vacuum expectation values (1-point functions)?
- HR for higher than 2-point functions
- Ward identities ?

Open Questions

- need gauge-invariant variables to higher order (2^{nd} order done)
- Lagrangian contains more than two derivatives: Boundary value problem?

Outlook

Other Possible Applications

- backgrounds with flavour
- black hole backgrounds (finite temperature)
- non-relativistic AdS/CFT

Thank you