Holographic flavors in Chern-Simons-matter theories

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INFN-Spain 2012
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Based on work with E. Conde, N. Jokela, J. Mas, D. Zoakos
(1105.6045, 1211.0630)

Plan of the talk

- Review of the ABJM model
- Addition of flavor
- Backreacted flavored backgrounds
- Massive probes
- Flavored black hole
- Brane thermodynamics

ABJM Chern-Simons-matter theories

(Aharony et al. 0806.1218)

Associated to M2-branes in $\mathbb{C}^4/\mathbb{Z}_k$ in M-theory

Field Theory Chern-Simons-matter theories in 2+1 dimensions

gauge group: $U(N)_k \times U(N)_{-k}$

Field content (bosonic)

-Two gauge fields $A_\mu,\ \hat{A}_\mu$

-Four complex scalar fields: $C^I\ \ (I = 1, \cdots, 4)$ bifundamentals $(N, \bar{N})$

Action

$$S = k\, CS[A] - k\, CS[\hat{A}] - k\, D_\mu C^{I\,\dagger} D^\mu C^I - V_{\text{pot}}(C)$$

$V_{\text{pot}}(C) \rightarrow$ sextic scalar potential

The ABJM model has $\mathcal{N} = 6$ SUSY in 3d

It has two parameters

$N \to$ rank of the gauge groups

$k \to$ CS level $(1/k \sim$ gauge coupling$)$

't Hooft coupling $\lambda \sim \frac{N}{k}$

It is a CFT in 3d with very nice properties

-partition function and Wilson loops can be obtained from localization! (Mariño, Putrov, Drukker)

-has many integrability properties (Bethe ansatz, Wilson loop/amplitude relation, ...)

-connection to FQHE?

It is the 3d analogue of N=4 SYM

M-theory description for large N $\rightarrow$ $\boxed{AdS_4 \times \mathbb{S}^7/\mathbb{Z}_k}$

Sugra description in type IIA

Represent S^7 as a U(1) bundle over $\mathbb{CP}^3$

Reduce from 11d to 10d along the $U(1)$ fiber φ $\rightarrow$ $AdS_4 \times \mathbb{CP}^3 +$ fluxes

$$\mathbb{CP}^3 = \mathbb{C}^4/(z_i \sim \lambda z_i)$$

$$\boxed{ds^2 = L^2\, ds^2_{AdS_4} + 4\, L^2\, ds^2_{\mathbb{CP}^3}} \qquad L^4 = 2\pi^2 \frac{N}{k}$$

$$ds^2_{\mathbb{CP}^3} \rightarrow \text{Fubini-Study metric} \qquad ds^2_{AdS_4} = r^2\, dx^2_{1,2} + \frac{dr^2}{r^2}$$

$$\boxed{\begin{array}{ll} F_2 = 2k\, J & F_4 = \dfrac{3\pi}{\sqrt{2}} \left(kN \right)^{\frac{1}{2}} \Omega_{AdS_4} \\ e^{\phi} = \dfrac{2L}{k} = 2\sqrt{\pi} \left(\dfrac{2N}{k^5} \right)^{\frac{1}{4}} & \end{array}}$$

$J \rightarrow$ Kahler form of $\mathbb{CP}^3$

$\Omega_{AdS_4} \rightarrow$ volume form of AdS_4

Effective description for $N^{\frac{1}{5}} << k << N$

Flavor in Chern-Simons-matter systems in 2+1

Hohenegger&Kirsch 0903.1730

Gaiotto&Jafferis 0903.2175

Flavor branes (massless quarks)

D6-branes extended in AdS_4 and wrapping $\mathbb{RP}^3 \subset \mathbb{CP}^3$

Introduce quarks in the $(N,1)$ and $(1,N)$ representation

$$Q_1 \rightarrow (N,1) \qquad Q_2 \rightarrow (1,N) \qquad \tilde{Q}_1 \rightarrow (\bar{N},1) \qquad \tilde{Q}_2 \rightarrow (1,\bar{N})$$

coupling to the vector multiplet

$$Q_1^{\dagger}\, e^{-V}\, Q_1 \,+\, Q_2^{\dagger}\, e^{-\hat{V}}\, Q_2 \,+\, \text{antiquarks}$$

$V,\, \hat{V}$ vector supermultiplets for $A\,,\,\hat{A}$

coupling to the bifundamentals

$\longrightarrow$ $C^I \;=\; (A_1, A_2, B_1^{\dagger}, B_2^{\dagger})$

$$\tilde{Q}_1\, A_i B_i\, Q_1 \;, \qquad \tilde{Q}_2\, B_i\, A_i\, Q_2$$

plus quartic terms in $Q, \tilde{Q}$'s

Backreaction

$$S_{WZ} = T_{D_6} \sum_{i=1}^{N_f} \int_{\mathcal{M}_7^{(i)}} \hat{C}_7 \to T_{D_6} \int_{\mathcal{M}_{10}} C_7 \wedge \Omega$$

Ω is a charge distribution 3-form

Modified Bianchi identity

$$\boxed{dF_2 \; = \; 2\pi \; \Omega}$$

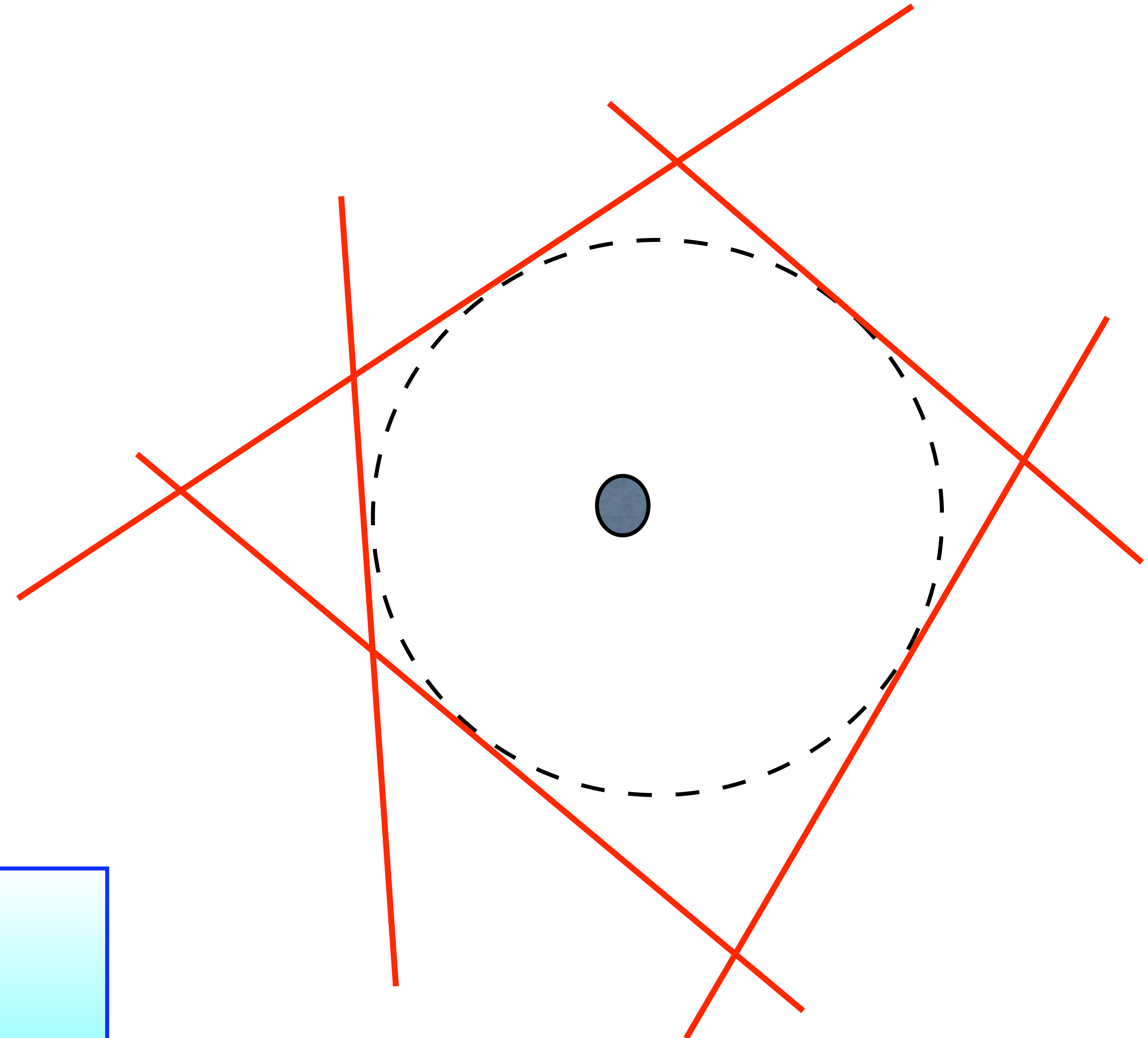
Localized solution in 11d for coincident massless flavors

$AdS_4 \times \mathcal{M}_7$ with $\mathcal{M}_7$ a hyperkahler 3-Sasakian manifold

$\mathcal{N} = 3$ with $U(N_f)$ flavor symmetry

One can keep conformality with flavor!

Smeared sources



-no delta-function sources

-still can preserve (less) SUSY

-much simpler (analytic) solutions

$-\text{flavor symmetry} : U(1)^{N_f}$

how can one find these delocalized solutions?

Backreaction with smearing

(E. Conde and AVR)

Write $\mathbb{CP}^3$ as an $\mathbb{S}^2$-bundle over $\mathbb{S}^4$

$$\boxed{ds^2_{\mathbb{CP}^3} = \frac{1}{4}\Big[ds^2_{\mathbb{S}^4} + \big(dx^i + \epsilon^{ijk} A^j x^k\big)^2\Big]}$$

$$\sum_i (x^i)^2 = 1$$

Fubini-Study metric

$A^i \rightarrow SU(2)$ instanton on $\mathbb{S}^4$

The RR two-form F_2 can be written as:

$$\boxed{F_2 = \frac{k}{2}\Big(E^1 \wedge E^2 - (\mathcal{S}^4 \wedge \mathcal{S}^3 + \mathcal{S}^1 \wedge \mathcal{S}^2)\Big)}$$

$$\frac{1}{2\pi}\int_{\mathbb{CP}^1} F_2 = k$$

$\mathcal{S}^i \rightarrow$ (rotated) basis of one-forms along $\mathbb{S}^4$

$E^i \rightarrow$ one-forms along the $\mathbb{S}^2$ fiber

Some Killing spinors are constant in this basis $\Longrightarrow$ deform to preserve them

Prescription: squash F_2 and the metric

$$\boxed{F_2 = \frac{k}{2}\left[\, E^1 \wedge E^2 - \eta\,(\mathcal{S}^4 \wedge \mathcal{S}^3 + \mathcal{S}^1 \wedge \mathcal{S}^2)\,\right]}$$

Induces violation of Bianchi identity

Massless Flavors

$$\boxed{\eta \equiv 1 + \frac{3N_f}{4k}}$$

Deformation parameter

$$\hat{\epsilon} = \frac{3}{4}\frac{N_f}{k} = \frac{3}{4}\frac{N_f}{N}\lambda$$

Flavored metric

$$\boxed{\begin{aligned} ds^2 &= L^2\, ds^2_{AdS_4} + ds^2_6 \\ ds^2_6 &= \frac{L^2}{b^2}\left[\, q\, ds^2_{\mathbb{S}^4} + \left(dx^i + \epsilon^{ijk} A^j x^k\right)^2 \right] \end{aligned}}$$

$$\left.\begin{aligned} &q \rightarrow \ \mathbb{C}\,\mathbb{P}^3 \text{ internal squashing} \\ &b \rightarrow \text{ relative } AdS_4/\mathbb{C}\,\mathbb{P}^3 \text{ squashing} \end{aligned}\right\} \Longrightarrow \text{constants}$$

SUSY with calibrations

$\tilde{\mathcal{K}} \longrightarrow$ 3-form for the D2's

$\mathcal{K} \longrightarrow$ 7-form for the D6's

$\mathcal{N} = 1$ BPS equations

$$\boxed{{}^*F_2 = -d\left(e^{-\phi}\,\mathcal{K}\right) \qquad d\left(e^{-\phi}\,h^{-\frac{1}{2}}\,{}^*\mathcal{K}\right) = 0 \qquad F_4 = -d(e^{-\phi}\,\tilde{\mathcal{K}})}$$

RR 7-form potential

$$\boxed{C_7 = e^{-\phi}\,\mathcal{K}}$$

$\mathcal{N} = 1$ superconformal SUSY implies

$$q^2 - 3(1+\eta)\,q + 5\eta = 0$$

$$\boxed{q = 3 + \frac{3}{2}\,\hat{\epsilon} - 2\sqrt{1 + \hat{\epsilon} + \frac{9}{16}\,\hat{\epsilon}^2}}$$

Also

$$\boxed{b = \frac{4 + \frac{13}{4}\,\hat{\epsilon} - \sqrt{1 + \hat{\epsilon} + \frac{9}{16}\,\hat{\epsilon}^2}}{3 + 2\hat{\epsilon}}}$$

limiting values

$$q \to \frac{5}{3} \qquad b \to \frac{5}{4} \qquad \text{as } N_f \to \infty$$

The new AdS_4 radius is:

$$\boxed{L^4 = 2\pi^2\,\frac{N}{k}\,\frac{(2-q)\,b^4}{q(q + \eta q - \eta)}}$$

Dilaton and F_4:

$$\boxed{e^{-\phi} = \frac{b}{4}\,\frac{\eta+q}{2-q}\,\frac{k}{L}}$$

$$\boxed{F_4 = \frac{3kb}{4}\,\frac{\eta+q}{2-q}\,L^2\,\Omega_{AdS_4}}$$

Regime of validity

$$L >> 1\,, \qquad e^{\phi} << 1$$

If $N_f/k \sim 1$ $\Longrightarrow$ $\boxed{N^{\frac{1}{5}} << k << N}$ (same as in the unflavored case)

When $N_f >> k$

$$L^4 \sim \frac{N}{N_f} \qquad e^{\phi} \sim \left(\frac{N}{N_f^5}\right)^{\frac{1}{4}} \Longrightarrow \boxed{N^{\frac{1}{5}} << N_f << N}$$

Flavor effects

Free energy on the 3-sphere (measures # dof's)

$$F(\mathbb{S}^3) = -\log |\, Z_{\mathbb{S}^3} \,| \Longrightarrow \boxed{F(\mathbb{S}^3) = \frac{\pi L^2}{2G_N}} \Longleftarrow \frac{1}{G_N} = \frac{1}{G_{10}}\, e^{-2\phi}\, Vol(\mathcal{M}_6)$$

In flavored ABJM

$$\boxed{F(\mathbb{S}^3) = \frac{\pi\sqrt{2}}{3}\, k^{\frac{1}{2}}\, N^{\frac{3}{2}}\, \xi\Big(\frac{N_f}{k}\Big)}$$

$$\xi\Big(\frac{N_f}{k}\Big) \equiv \frac{1}{16}\, \frac{q^{\frac{5}{2}}\,(\eta+q)^4}{(2-q)^{\frac{1}{2}}\,(q+\eta q-\eta)^{\frac{7}{2}}}$$

For small N_f/k

$$\xi = 1 + \frac{3}{4}\frac{N_f}{k} - \frac{9}{64}\Big(\frac{N_f}{k}\Big)^2 + \mathcal{O}\Big(\Big(\frac{N_f}{k}\Big)^3\Big)$$

$$\boxed{F(\mathbb{S}^3) = \frac{\pi\sqrt{2}}{3}\frac{N^2}{\sqrt{\lambda}} + \frac{\pi\sqrt{2}}{4} N_f\, N\sqrt{\lambda} - \frac{3\pi\sqrt{2}}{64} N_f^2\, \lambda^{\frac{3}{2}} + \cdots}$$

unflavored term $\sim N^{\frac{3}{2}}$ $\Longrightarrow$ amazing field theory match by Drukker et al. (1007.3837) !

For large N_f/k $\Longrightarrow$

$$\xi \sim \frac{225}{256}\sqrt{\frac{5}{2}}\sqrt{\frac{N_f}{k}} \approx 1.389\sqrt{\frac{N_f}{k}}$$

Comparison with 3-Sasakian $(U(N_f), \mathcal{N}=3$ flavors$)$

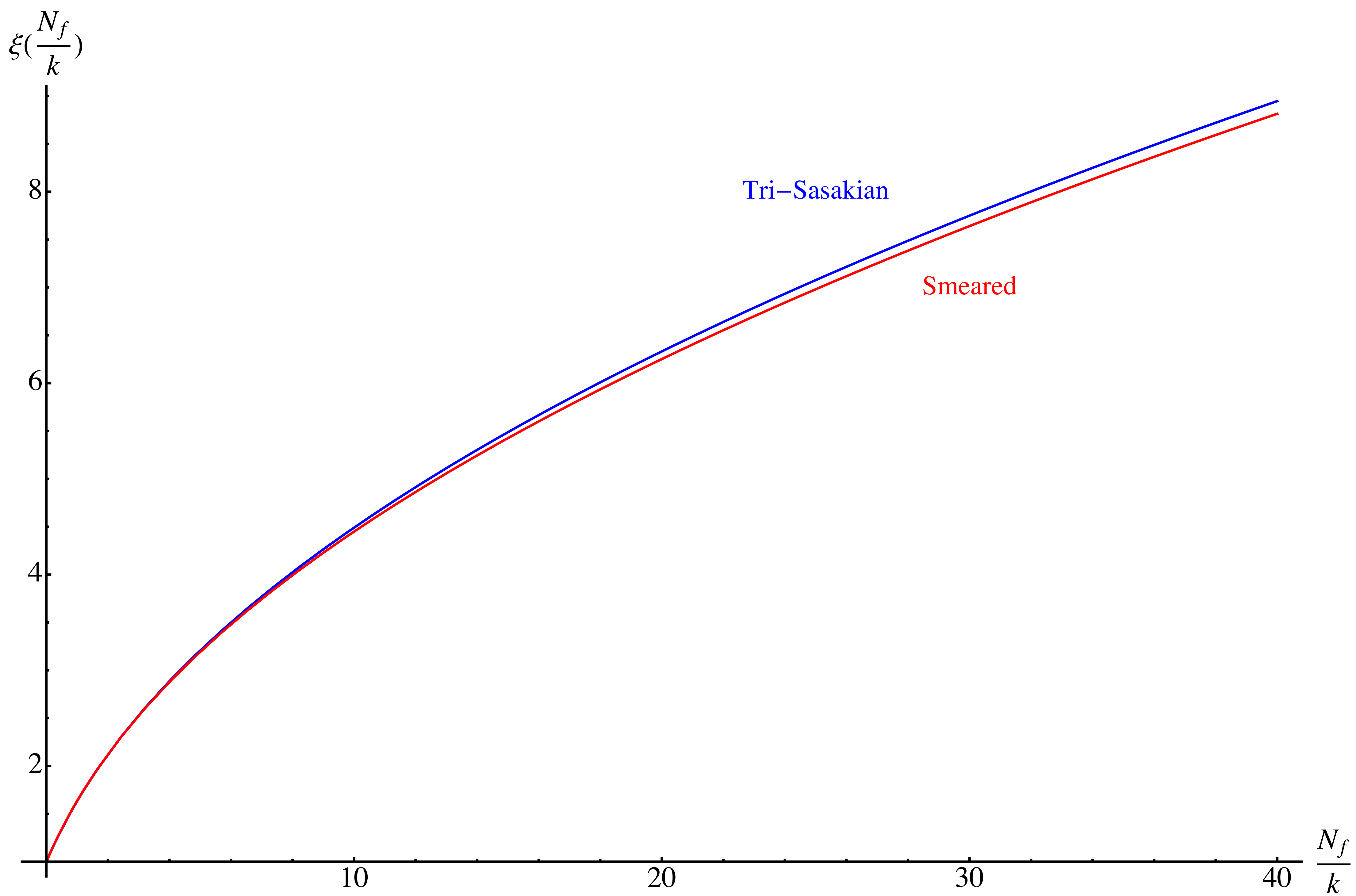
$$\boxed{\xi^{3-S} = \frac{1+\frac{N_f}{k}}{\sqrt{1+\frac{N_f}{2k}}}} \Longrightarrow$$

(Gaiotto&Jafferis 0903.2175)

$$\xi^{3-S} = 1 + \frac{3}{4}\frac{N_f}{k} - \frac{5}{32}\left(\frac{N_f}{k}\right)^2 + \mathcal{O}\left(\left(\frac{N_f}{k}\right)^3\right)$$

$$\xi^{3-S} \sim \sqrt{2}\sqrt{\frac{N_f}{k}} \qquad \text{when } N_f/k \to \infty$$

Field theory match: Couso-Santamaria et al. 1011.6281



$\xi(\frac{N_f}{k})$
8
6
4
2
10
20
30
40
$\frac{N_f}{k}$
Tri−Sasakian
Smeared

quark-antiquark energy

$$\boxed{V_{q\bar{q}} = -\frac{Q}{d}} \qquad Q = \frac{4\pi^2 L^2}{\left[\Gamma\left(\frac{1}{4}\right)\right]^4}$$

(Maldacena, Rey)

In ABJM with flavor

$$\boxed{Q = \frac{4\pi^3\sqrt{2\lambda}}{\left[\Gamma\left(\frac{1}{4}\right)\right]^4}\,\sigma} \longleftarrow \sigma = \frac{1}{4}\,\frac{q^{\frac{3}{2}}\,(\eta+q)^2\,(2-q)^{\frac{1}{2}}}{(q+\eta q-\eta)^{\frac{5}{2}}}$$

Series expansion

$$\sigma = 1 - \frac{3}{8}\,\frac{N_f}{k} + \frac{9}{64}\left(\frac{N_k}{k}\right)^2 + \cdots$$

Dynamical quarks screen the Coulomb interaction

Flavor brane probes in flavored ABJM

→ D6 extended in x^μ, r, $\mathbb{RP}^3$

$$\mathbb{RP}^3 \longrightarrow \begin{cases} \text{-2 directions inside } \mathbb{S}^4 \\ \text{-1 direction inside } \mathbb{S}^2 \end{cases}$$

Embedding → Write the $\mathbb{S}^2$ metric as $ds^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

D6 $\rightarrow$ extended in φ with a profile $\theta(r)$

New Cartesian-like coordinates

$$\boxed{R = r^b \cos\theta \qquad \rho = r^b \sin\theta}$$

$b \rightarrow$ relative $AdS_4/\mathbb{CP}^3$ squashing

(θ, r) metric → $\frac{L^2}{b^2(\rho^2 + R^2)}\left[\, d\rho^2 + dR^2 \,\right]$

DBI+WZ action $\Longrightarrow$

$$S_{DBI} = -T_{D6}\int d^7\zeta\, e^{-\phi}\sqrt{-\det \hat{g}_7} = \int d^3x\, d\rho\, \mathcal{L}_{DBI}$$

$$S_{WZ} = T_{D6}\int \hat{C}_7 = T_{D6}\int e^{-\phi}\hat{\mathcal{K}} = \int d^3x\, d\rho\, \mathcal{L}_{WZ}$$

Embeddings parametrized by $R(\rho)$

$$\boxed{\mathcal{L} \sim \rho\left[\rho^2 + R^2\right]^{\frac{3}{2b}-1}\left(\sqrt{1+R'^2} - 1\right)}$$

SUSY solution $\Longrightarrow$ $R = \text{constant}$ $\Longrightarrow$ $\mathcal{L} = 0$

arbitrary solution $\Longrightarrow$ $\mathcal{L} \sim \rho^{1-\frac{3}{b}}$ $\Longrightarrow$ finite action

Depends on the gauge for C_7!!

$C_7 \rightarrow C_7 + d\Lambda_6$ generates boundary conterterms

gauge for $C_7 \sim$ scheme in holographic renormalization

General asymptotic behavior

$$R \sim m + \frac{c}{r^{3-2b}}$$

Compare with

$$\phi \sim \phi_0 \, r^{\Delta-3} + \frac{\langle \mathcal{O} \rangle}{r^{\Delta}}$$

ϕ_0 is the source of $\mathcal{O}$

$\Delta \longrightarrow$ dimension of $\mathcal{O}$

$$\boxed{\Delta = 3 - b}$$

same result from the normalizable fluctuations of the scalars transverse to the D6-branes

In our case $\mathcal{O} \sim \bar{\psi}\psi$

$$\boxed{\dim(\bar{\psi}\psi) = 3 - b}$$

$$\dim(\bar{\psi}\psi) = 2 - \frac{3}{16}\frac{N_f}{k} + \frac{63}{512}\left(\frac{N_f}{k}\right)^2 + \cdots$$

$$\dim(\bar{\psi}\psi) \to \frac{7}{4} \qquad \left(\frac{N_f}{k} \to \infty\right)$$

Mass anomalous dimension $\gamma_m = 3 - \Delta = b$

Flavored black hole

$$\boxed{ds^2 \;=\; L^2 \; ds^2_{BH_4} \;+\; ds^2_6}$$

$$ds^2_{BH_4} = -r^2 h(r) dt^2 + \frac{dr^2}{r^2 h(r)} + r^2 \left[(dx^1)^2 + (dx^2)^2\right] \qquad\qquad h(r) \;=\; 1 \;-\; \frac{r_h^3}{r^3}$$

Temperature

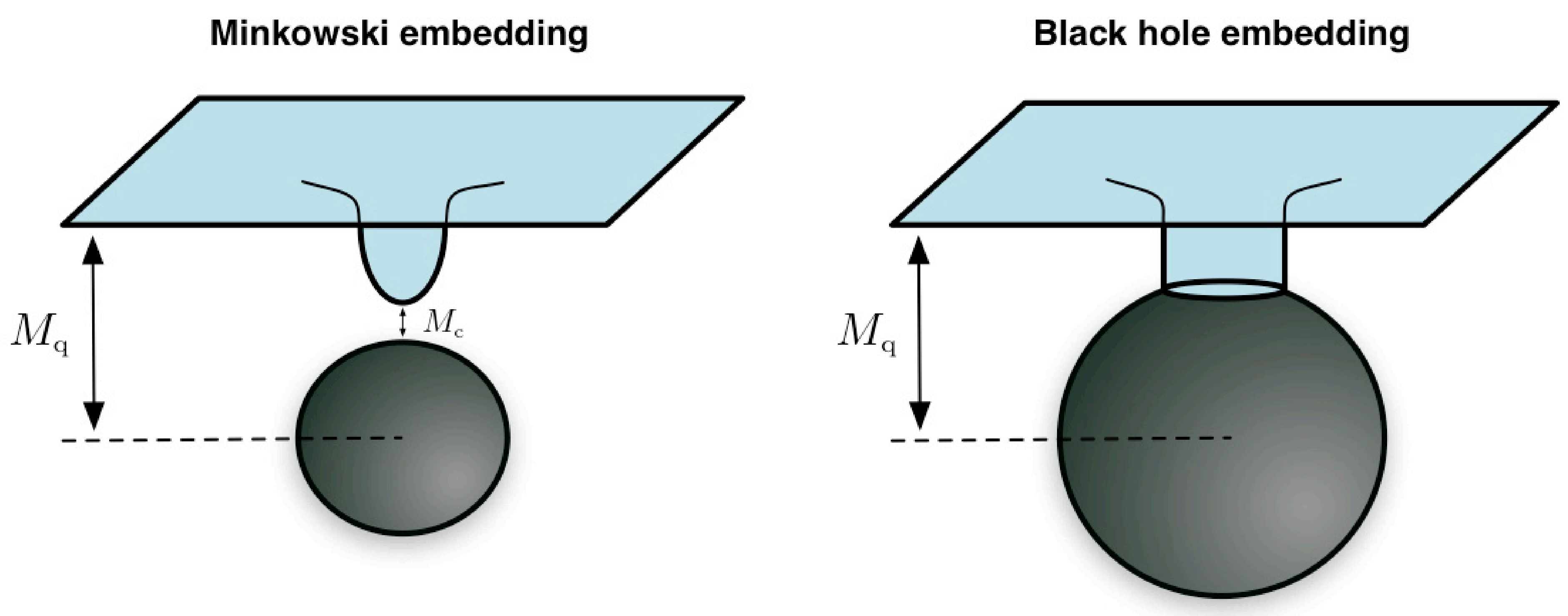
$$\boxed{T \;=\; \frac{3\, r_h}{4\pi}}$$

Free energy and entropy

$$\boxed{F_{back} = -\frac{1}{9}\left(\frac{4\pi}{3}\right)^2 \frac{N^2}{\sqrt{2\lambda}}\, \xi\left(\frac{N_f}{k}\right) T^3} \qquad \boxed{s_{back} = \frac{1}{3}\left(\frac{4\pi}{3}\right)^2 \frac{N^2}{\sqrt{2\lambda}}\, \xi\left(\frac{N_f}{k}\right) T^2}$$

Probes in the flavored BH

We add massive quarks



Low T

High T

Meson melting transition → first-order phase transition

(Babington et al. 0306018, Mateos et al. 0701132,...)

Embeddings governed by the DBI+WZ action

WZ action $\longrightarrow$ $$S_{WZ} = T_{D6} \int \hat{C}_7$$

C_7 has to be improved to get a consistent thermodynamics

$$C_7 = e^{-\phi}\,\mathcal{K} + \delta C_7$$

$$e^{-\phi}\,\hat{\mathcal{K}} = \frac{L^7 q}{b^3}\, e^{-\phi}\, d^3x \wedge \Big[\, \frac{r^3}{b} \sin\theta\, \cos\theta\, d\theta + r^2 \sin^2\theta dr \Big] \wedge \Xi_3$$

Represent δC_7 as

$$\delta C_7 = \frac{L^7 q}{b^3}\, e^{-\phi}\, d^3x \wedge \Big[L_1(\theta)\, d\theta + L_2(r)\, dr\Big] \wedge \Xi_3 \qquad\qquad d(\delta C_7) = 0$$

The angular part of C_7 must vanish at the horizon (Jensen 1006.3066)

$$L_1(\theta) = -\frac{r_h^3}{b} \sin\theta \cos\theta$$

Total action

$$S = -\mathcal{N}_r \int d^3x\, dr\, r^2 \,\sin\theta \Big[\sqrt{1 + \frac{r^2\, h(r)}{b^2}\;\dot\theta^2} - \sin\theta - \frac{r h(r)}{b}\,\cos\theta\,\dot\theta \Big] + \mathcal{N}_r\, r_h^3 \int d^3\, x\, \Delta_0$$

with

$$\boxed{\int dr\, L_2(r) \equiv r_h^3\, \Delta_0} \longrightarrow$$ zero-point energy

$$\boxed{\mathcal{N}_r \,=\, \frac{1}{4\sqrt{2}\,\pi}\,\frac{N^{\frac{3}{2}}}{\sqrt{k}}\,\zeta\Big(\frac{N_f}{k}\Big)}$$

$$\boxed{\zeta\Big(\frac{N_f}{k}\Big) \equiv \frac{1}{32}\,\frac{\sqrt{2-q}\,(\eta+q)^5\, q^{\frac{7}{2}}}{(q+\eta q-\eta)^{\frac{11}{2}}} = \frac{b^3}{q}\,\sigma} \longrightarrow$$ determine the size of the cycle

It satisfies

$$\frac{\partial \mathcal{L}}{\partial \dot\theta}\Big|_{r=r_h} = 0 \longrightarrow$$ generalized momentum vanishes at the horizon

Free energy density

$$F = T\, S_E \longrightarrow F = -\frac{S}{\int d^3x}$$

Decoupling of quarks with infinite mass

$$\lim_{m\to\infty} F = \left(\frac{1}{4b} - \Delta_0\right) r_h^3 \mathcal{N}_r$$

it should vanish

$$\boxed{\Delta_0 \; = \; \frac{1}{4b}}$$

non-trivial test → massless flavors → background and probe flavors are of the same type

expand the flavor function ξ as

$$\xi\left(\frac{N_f+1}{k}\right) = \xi\left(\frac{N_f}{k}\right) + \xi'\left(\frac{N_f}{k}\right)\frac{1}{k} + \cdots \quad \longrightarrow \quad \Delta F_{back} \; = - \left(\frac{4\pi}{3}\right)^2 \frac{N^2}{9\sqrt{2\lambda}} \, \frac{1}{k} \, \xi'\left(\frac{N_f}{k}\right) T^3$$

$$\Delta F_{back} \; = \; F(m=0) \quad \longleftrightarrow \quad \boxed{\xi' \; = \; \frac{3}{4b}\,\zeta}$$

→ It is true (non-trivially!)

background+probe entropy

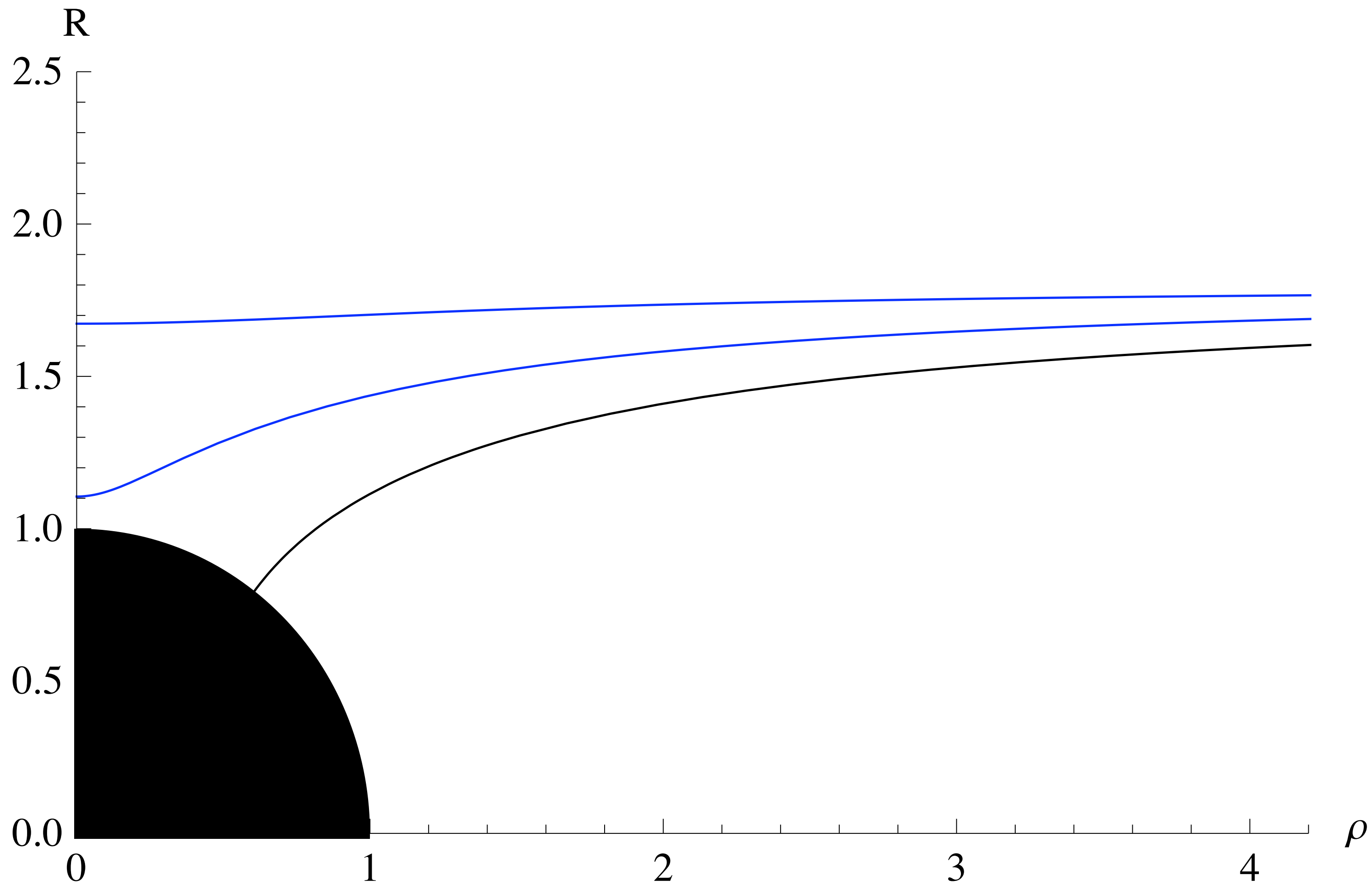
$$s_{total} \; = \; s_{back} + s \approx \frac{1}{3}\left(\frac{4\pi}{3}\right)^2 \frac{N^2}{\sqrt{2\lambda}}\,\xi\left(\frac{N_f+1}{k}\right) T^2 \,, \qquad\qquad (m\to 0)$$

massless probe entropy $\sim$ increase of the area of the horizon

Mass and condensate

$$\boxed{R \sim m + \frac{c}{\rho^{\frac{3}{b}-2}}}$$

$$\rho \to \infty$$

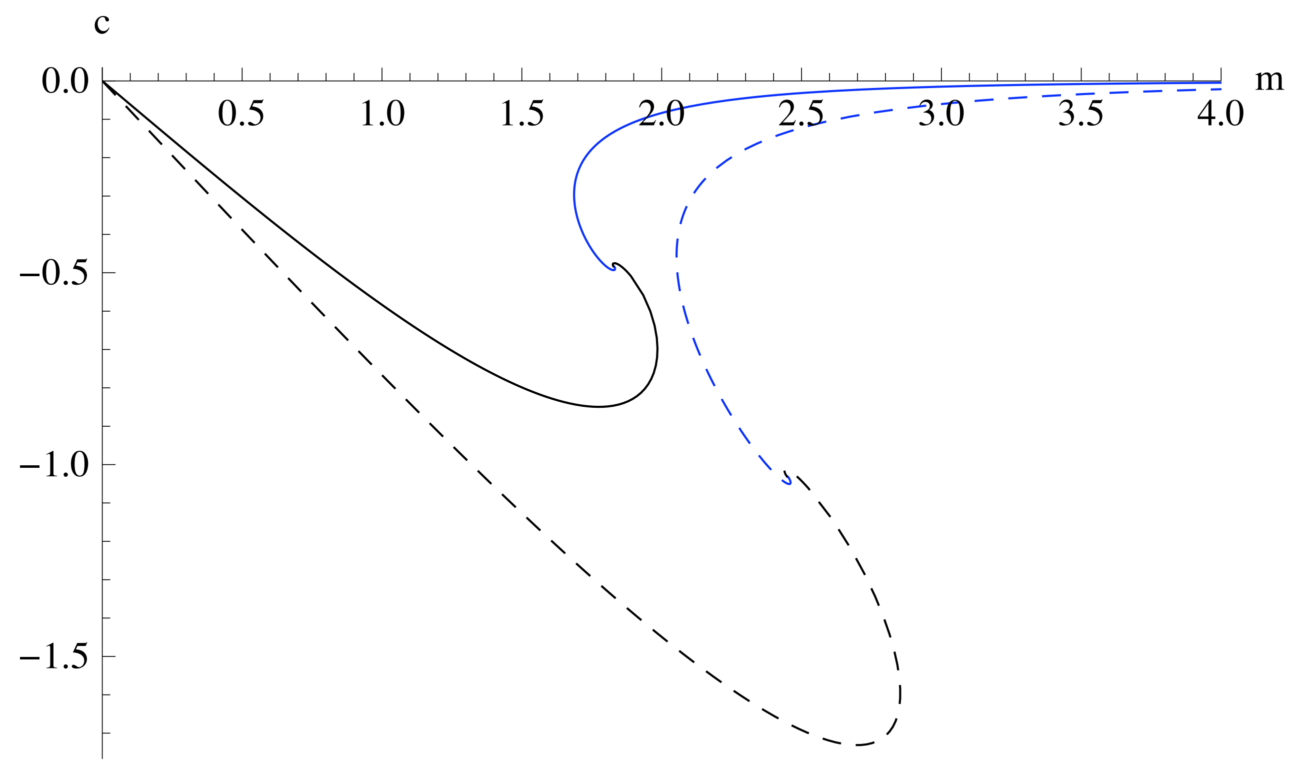


Dictionary

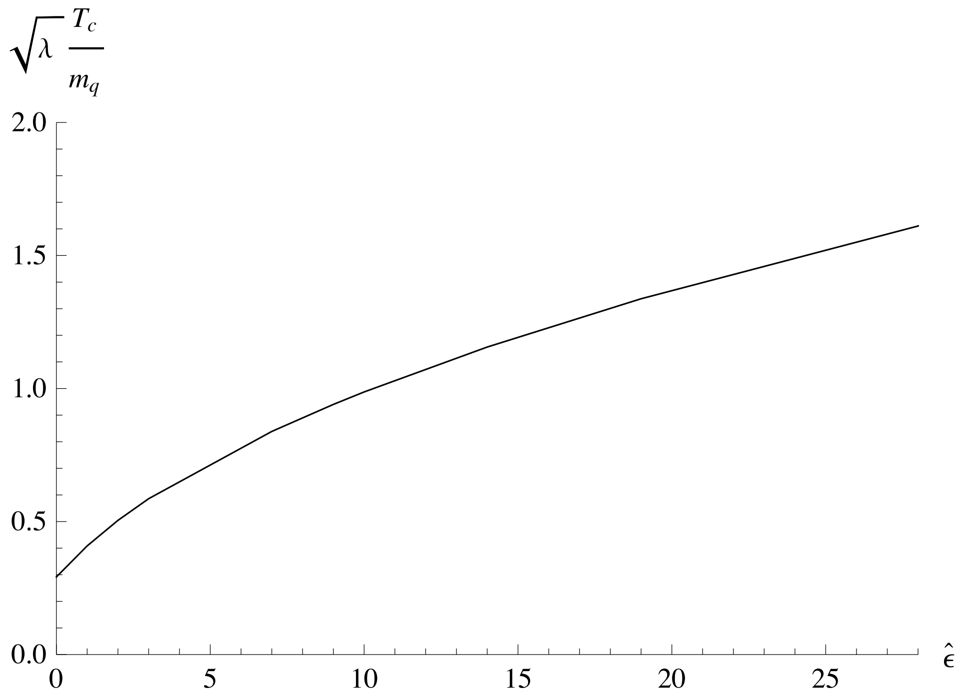
$$\boxed{m_q = \frac{2^{\frac{1}{3}}\pi}{3}\,\sqrt{2\lambda}\,\sigma T m^{\frac{1}{b}}}$$

$$\boxed{\langle \mathcal{O}_m \rangle = -\frac{2^{\frac{2}{3}}\,\pi}{9}\,\frac{(3-2b)\,b^2}{q}\,\left(\frac{3m_q}{2^{\frac{1}{3}}\pi\,\sqrt{2\lambda}\,\sigma}\,\frac{1}{T}\right)^{b-1}\,N\,T^2\,c}$$

Numerical $c = c(m)$ $\Rightarrow$



Phase transition temperature



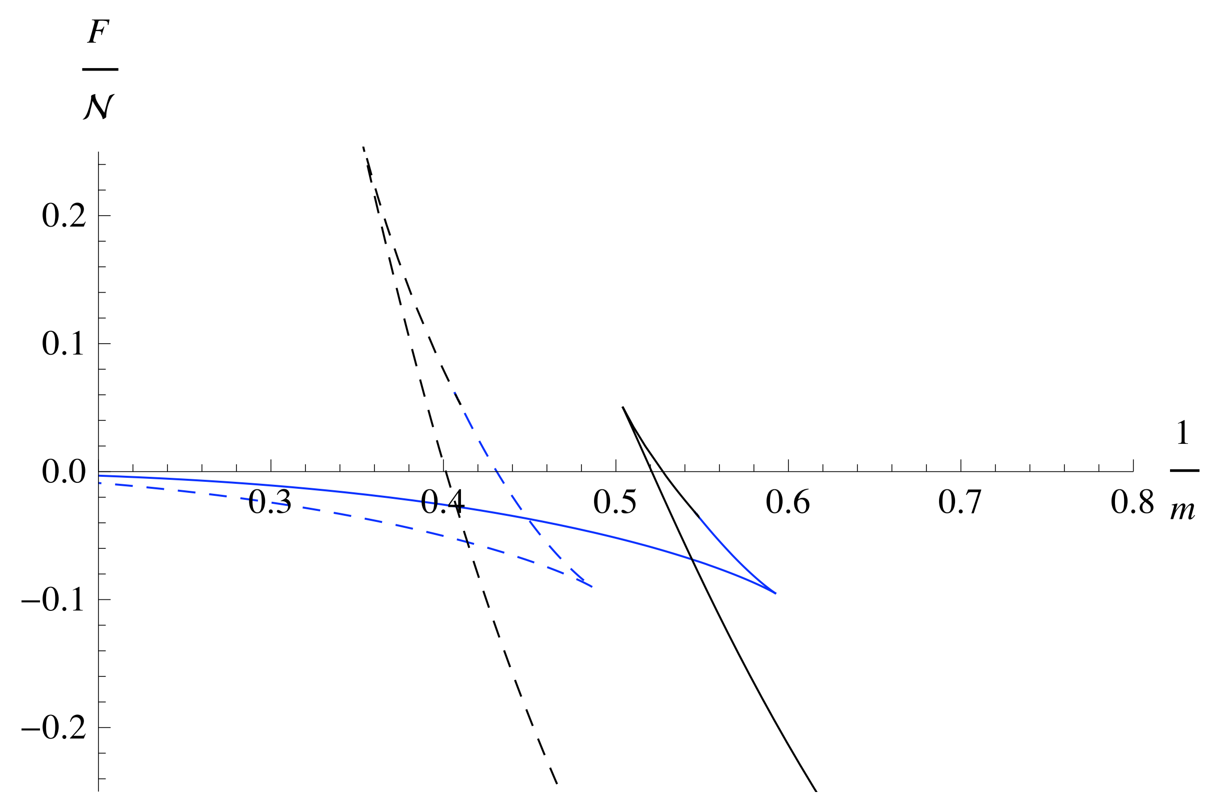
$$\Rightarrow \quad \sqrt{\lambda}\,\frac{T_c}{m_q} \propto \sqrt{N_f}$$

Free energy density

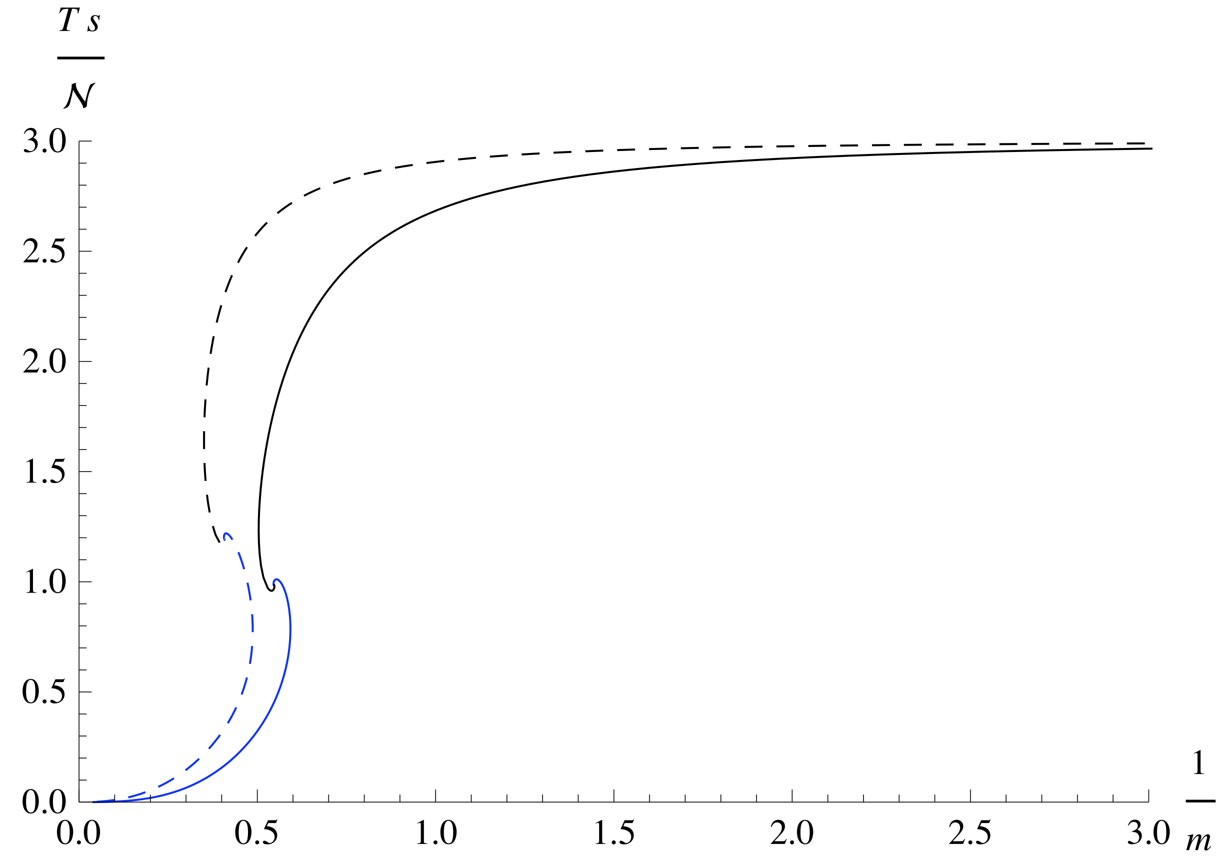
$$\boxed{F \approx -\mathcal{N}} \qquad m \longrightarrow 0$$

$$\mathcal{N} = \left(\frac{4\pi}{3}\right)^3 \frac{\mathcal{N}_r}{4b}\, T^3$$

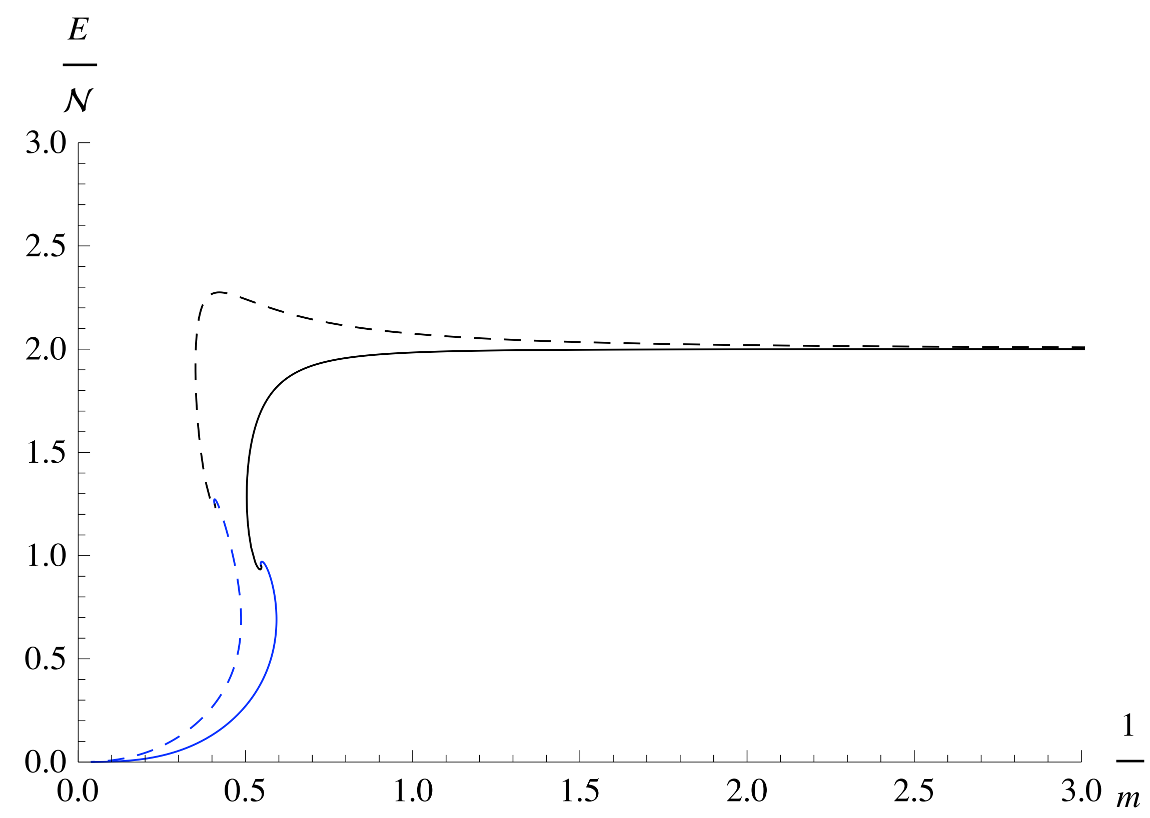
swallow tail behavior near the transition $\Rightarrow$



Entropy

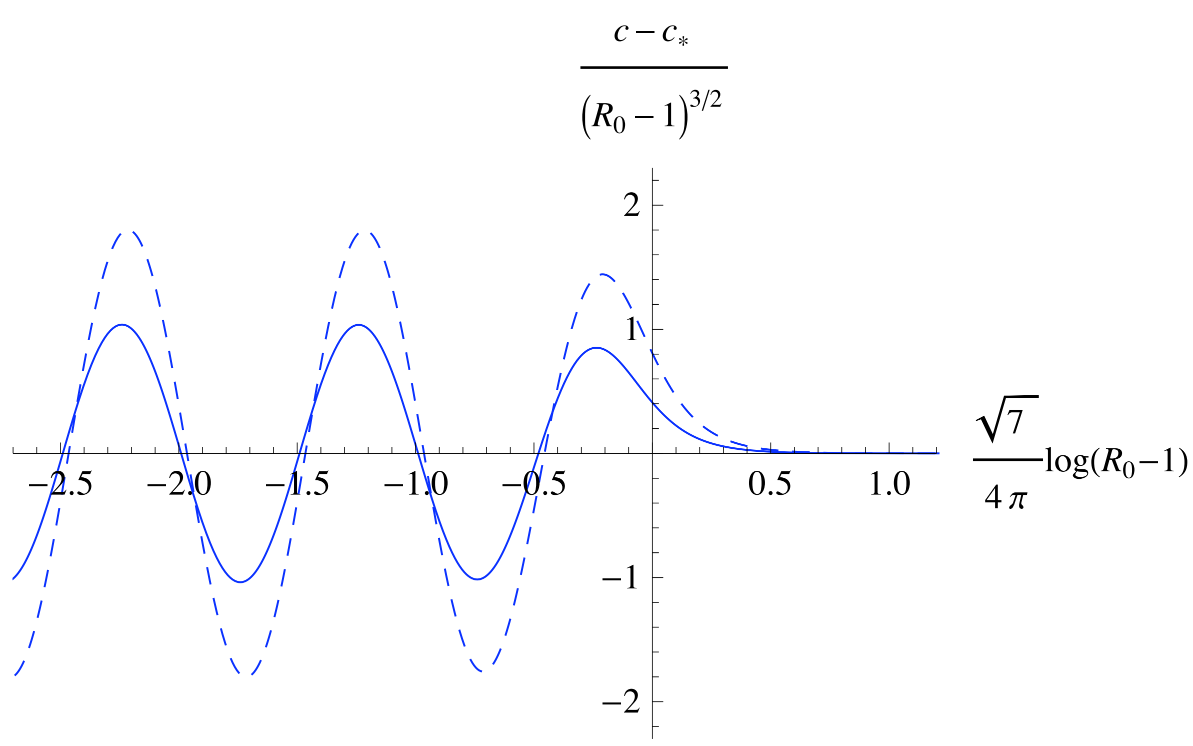


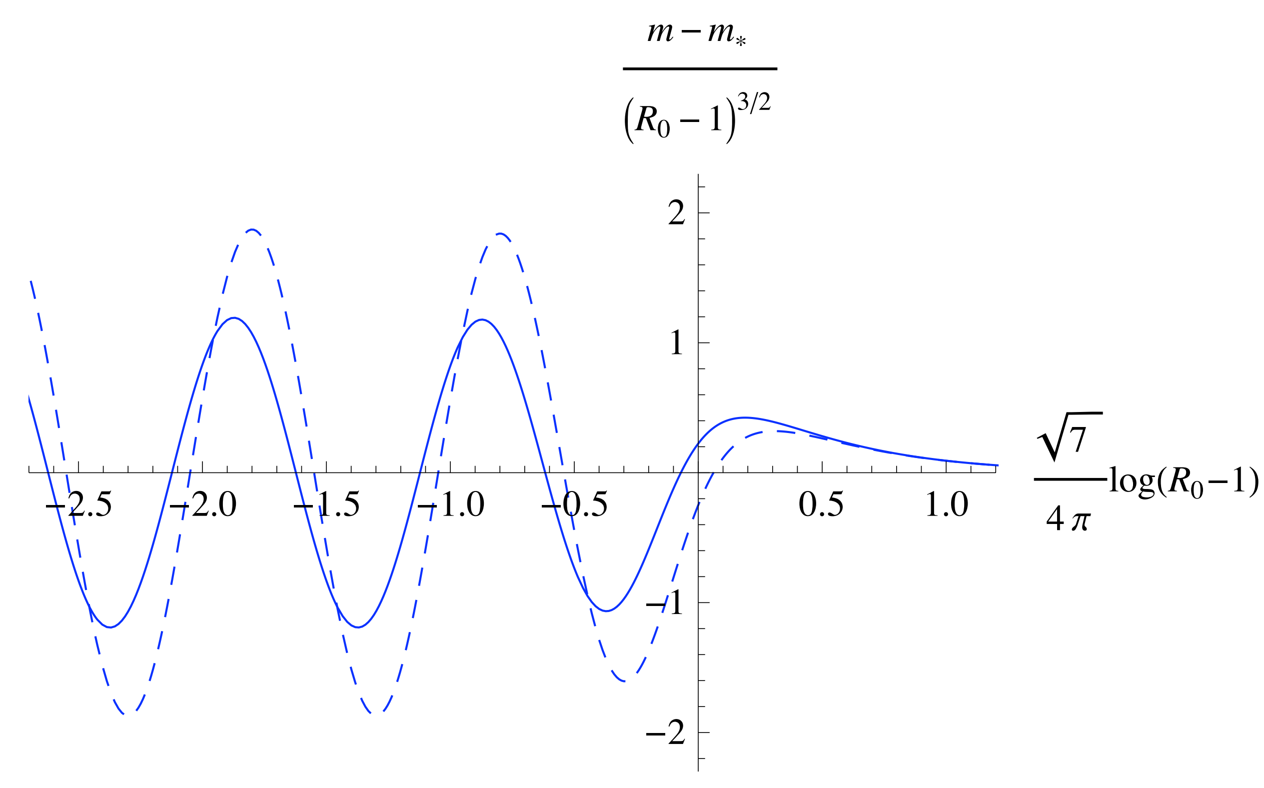
Internal energy



Near the transition

- self-similar behavior
- c and m oscillate






Thank You!