Supermembrane interaction with dynamical D=4 N=1 supergravity

Superfield Lagrangian description and spacetime equations of motion

Igor A. Bandos †♣ and Carlos Meliveo†

† Department of Theoretical Physics, University of the Basque Country, Bilbao, Spain,
♣ IKERBASQUE, the Basque Foundation for Science, Bilbao, Spain

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Outline

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- $*\hat{E}^a$ is the Hodge dual two form $*\hat{E}^a := \frac{1}{2} d\xi^m \wedge d\xi^n \sqrt{g}\epsilon_{mnk} g^{kl} \hat{E}_l^a$.

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$$C_3 = C_3(Z) = \tfrac{1}{3} dZ^{\underline{K}} \wedge dZ^{\underline{N}} \wedge dZ^{\underline{M}} C_{\underline{MNK}}(Z) := \tfrac{1}{3} E^{\underline{C}} \wedge E^{\underline{B}} \wedge E^{\underline{A}} C_{\underline{ABC}}(Z) \; ,$$

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- In flat superspace (SSP) $E^a = dX^a - i(d\theta\sigma^a\bar\theta - c.c.)$, $E^\alpha = d\theta^\alpha$, $C_3 = c_3$ such that $h_4 = dc_3 := -\frac{i}{4} E^b \wedge E^a \wedge E^\alpha \wedge E^\beta \sigma_{ab\alpha\beta} + c.c.$ is closed,

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- and the action possesses a 2-parametric local fermionic κ–symmetry

$$i_\kappa \hat{E}^a := \delta_\kappa \hat{Z}^M E_M{}^a(\hat{Z}) = 0 \; , \qquad i_\kappa \hat{E}^\alpha = \kappa^\alpha = \bar\kappa_{\dot\alpha} \bar{\tilde{\gamma}}^{\dot\beta\alpha} \; ,$$
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- one can obtain the supergravity superfield equations with the contributions of supermembrane supercurrent(s).

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- But, as we will see, the supermembrane coupling to *dynamical* supergravity implies an additional restriction on minimal supergravity supermultiplet.

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$$T^a := \mathcal{D}E^a = -2i\sigma^a_{\alpha\dot\alpha} E^\alpha \wedge \bar{E}^{\dot\alpha} - \tfrac{1}{8} E^b \wedge E^c \varepsilon^a{}_{bcd} G^d \; ,$$

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- These expressions for bosonic and fermionic torsion 2-forms involve *main superfields* $R = (\bar{R})^*$, and $G_a = (G_a)^*$, which obey ($G_{\alpha\dot{\alpha}} := G_a \sigma^a_{\alpha\dot{\alpha}}$)

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- Minimal supergravity constraints and their consequences can be collected in (see [Wess & Zumino 77, Grimm, Wess & Zumino 78])

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$$T_{\alpha\dot{\alpha}\ \beta\dot{\beta}\ \gamma} \equiv \sigma^a_{\alpha\dot{\alpha}} \sigma^b_{\beta\dot{\beta}} \epsilon_{\gamma\delta} T_{ab}{}^\delta = -\tfrac{1}{8} \epsilon_{\alpha\beta} \bar{\mathcal{D}}_{(\dot{\alpha}|} G_{\gamma|\dot{\beta})} - \tfrac{1}{8} \epsilon_{\dot{\alpha}\dot{\beta}} [W_{\alpha\beta\gamma} - 2\epsilon_{\gamma(\alpha} \mathcal{D}_{\beta)} R] \ .$$

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- It is symmetric, $W_{\alpha\beta\gamma} = W_{(\alpha\beta\gamma)}$, and chiral

$$\bar{\mathcal{D}}_{\dot{\alpha}} W^{\alpha\beta\gamma} = 0 \ , \qquad \mathcal{D}_\alpha \bar{W}^{\dot{\alpha}\dot{\beta}\dot{\gamma}} = 0 \ .$$

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- The superfield generalization of the Ricci tensor is

$$R_{bc}{}^{ac} = \frac{1}{32}(\mathcal{D}^{\beta}\bar{\mathcal{D}}^{(\dot{\alpha}|}G^{\alpha|\dot{\beta})} - \bar{\mathcal{D}}^{\dot{\beta}}\mathcal{D}^{(\beta}G^{\alpha)\dot{\alpha}})\sigma^{a}_{\alpha\dot{\alpha}}\sigma_{b\beta\dot{\beta}} - \\ -\frac{3}{64}(\bar{\mathcal{D}}\bar{\mathcal{D}}\bar{R} + \mathcal{D}\mathcal{D}R - 4R\bar{R})\delta^{a}_{b} \ .$$

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- and the superfield generalization of the *l.h.s.* of the supergravity Rarita–Schwinger equation reads
$\epsilon^{abcd} T_{bc}{}^{\alpha}\sigma_{d\alpha\dot\alpha} = \frac{i}{8}\tilde{\sigma}^{a\dot\beta\beta}\bar{\mathcal{D}}_{(\dot\beta|} G_{\beta|\dot\alpha)} + \frac{3i}{8}\sigma^a_{\beta\dot\alpha}\mathcal{D}^\beta R \;\; ,$

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- Indeed, these ⇒ 'free' SUGRA equations of motion

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Closed 4-form in the superspace of minimal SUGRA

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- The superspace of minimal supergravity allows for existence of two closed 4-forms

$$H_{4L} = -\frac{i}{4} E^b \wedge E^a \wedge E^\alpha \wedge E^\beta \sigma_{ab\ \alpha\beta} - \frac{1}{128} E^d \wedge E^c \wedge E^b \wedge E^a \epsilon_{abcd} R \,,$$
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- Indeed, $\delta S_{p=2} = -\frac{1}{2} \int\limits_{W^3} * \hat{E}_a \wedge \delta \hat{E}^a - \int\limits_{W^3} \delta \hat{C}_3$, and, when the variation are produced by $\delta \hat{Z}^M$ only, the variations of the 'potentials' are expressed in terms of 'contractions' of the field strengths superforms $\delta \hat{E}^a = i_\delta T^a$, $\delta \hat{C}_3 = i_\delta H_4$
- (with $i_\delta \hat{E}^A := \delta \hat{Z}^M E_M^A(\hat{Z})$, $\quad i_\delta(\Omega_p \wedge \Omega_q) := \Omega_p \wedge i_\delta \Omega_q + (-)^q i_\delta \Omega_p \wedge \Omega_q$ for any p- and q–forms).

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- Clearly $\delta E^a(Z)$ and $\delta C_3(Z)$ are not arbitrary as far as $E^a(Z)$ and $C_3(Z)$ obey the superspace SUGRA constraints. The basic variations are free parameters of the solution of the equations stating that the constraints are preserved by variation.

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- The admissible variations of supervielbein read [Wess & Zumino 78]

$$\delta E^a = E^a(\Lambda(\delta) + \bar{\Lambda}(\delta)) - \frac{1}{4}E^b\tilde{\sigma}_b^{\dot{\alpha}\alpha}[\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}]\delta H^a + iE^\alpha\mathcal{D}_\alpha\delta H^a - i\bar{E}^{\dot{\alpha}}\bar{\mathcal{D}}_{\dot{\alpha}}\delta H^a ,$$

$$\delta E^\alpha = E^a\Xi_a^\alpha(\delta) + E^\alpha\Lambda(\delta) + \frac{1}{8}\bar{E}^{\dot{\alpha}}R\sigma_{a\dot{\alpha}}{}^\alpha\delta H^a ,$$

where

$$2\Lambda(\delta) + \bar{\Lambda}(\delta) = \frac{1}{4}\tilde{\sigma}_a^{\dot{\alpha}\alpha}\mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot{\alpha}}\delta H^a + \frac{1}{8}G_a\delta H^a + 3(\mathcal{D}\mathcal{D} - \bar{R})\delta\mathcal{U}$$

and the explicit expression for $\Xi_a^\alpha(\delta)$ is not needed for our discussion.

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- The corresponding off–shell formulation of SUGRA which we call *special minimal SUGRA* was described by Siegel [78], Siegel and Gates [79] and Waldram and Ovrut [97]

Outline

Special minimal SUGRA. Dynamical generation of cosmological constant

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- In terms of component formulation, in special minimal SUGRA one of two auxiliary scalars of the generic minimal SUGRA [Stelle & West 78, Ferrara & van Nieuwenhuizen 78] is replaced by a divergence of an auxiliary vector, $S \mapsto \partial_\mu k^\mu$.

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- In terms of component formulation, in special minimal SUGRA one of two auxiliary scalars of the generic minimal SUGRA [Stelle & West 78, Ferrara & van Nieuwenhuizen 78] is replaced by a divergence of an auxiliary vector, $S \mapsto \partial_\mu k^\mu$.
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- This seemingly minor modification has drastic consequence already in the case of 'free' supergravity:
- it results in the dynamical generation of cosmological constant [the effect first described in superfield context by Ogievetsky and Sokatchev [1980]].

Dynamical generation of cosmological constant in sMin SUGRA

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- Substituting $(\mathcal{D}\mathcal{D}-\bar{R})\delta\mathcal{U}=\frac{1}{12}(\mathcal{D}\mathcal{D}-\bar{R})\left(i\delta V+\frac{1}{2}\bar{\mathcal{D}}_{\dot{\alpha}}\delta\bar{\kappa}^{\dot{\alpha}}\right)$, one finds

$$\delta S_{SG}=\frac{1}{6}\int d^8ZE\ \left[G_a\,\delta H^a+(R-\bar{R})i\delta V\right]-$$
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- $-\Lambda \propto c^2$, c is an arbitrary integration constant $\Rightarrow$ *cosmological constant is generated dynamically* in the special min SUGRA.

Special minimal SUGRA variations.

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- Resuming the special minimal SUGRA variation of the bosonic supervielbein and three form potential read (simplified):

$$\delta E^a = E^a(\Lambda(\delta)+\bar{\Lambda}(\delta)) - \frac{1}{4}E^b\tilde{\sigma}_b^{\dot{\alpha}\alpha}[\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}]\delta H^a + iE^\alpha\mathcal{D}_\alpha\delta H^a - i\bar{E}^{\dot{\alpha}}\bar{\mathcal{D}}_{\dot{\alpha}}\delta H^a \,,$$
$$\delta C_3 = \frac{1}{3!}E^C\wedge E^B\wedge E^A\beta_{ABC}(\delta)$$

where

$$2\Lambda(\delta)+\bar{\Lambda}(\delta) = \frac{1}{4}\tilde{\sigma}_a^{\dot{\alpha}\alpha}\mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot{\alpha}}\delta H^a + \frac{1}{8}G_a\delta H^a + i/4(\mathcal{D}\mathcal{D}-\bar{R})\delta V \,,$$

and

$$\beta_{\alpha\beta\gamma}(\delta)=0\,, \qquad \beta_{\alpha\beta\dot{\gamma}}(\delta)=0\,,$$
$$\beta_{\alpha\dot{\beta}a}(\delta) = i\sigma_{a\alpha\dot{\beta}}\delta V\,, \qquad \beta_{\alpha\beta a}(\delta) = -\sigma_{ab\;\alpha\beta}\delta H^b\,,$$
$$\beta_{\alpha ab}(\delta) = \frac{1}{2}\epsilon_{abcd}\sigma^c_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}}\delta H^d + \frac{1}{2}\sigma_{ab\;\alpha}{}^\beta D_\beta\delta V\,,$$
$$\beta_{abc}(\delta) = \frac{i}{8}\epsilon_{abcd}\left(\left(\bar{\mathcal{D}}\bar{\mathcal{D}}-1/2R\right)\delta H^d - c.c.\right) +$$
$$+\frac{1}{4}\epsilon_{abcd}G^d\delta V + \frac{1}{8}\epsilon_{abcd}\tilde{\sigma}^{d\dot{\gamma}\gamma}[\mathcal{D}_\gamma,\bar{\mathcal{D}}_{\dot{\gamma}}]\delta V\,.$$

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- Now we are ready to study the interacting system action $S_{SG} + S_{p=2}$.

Outline

Supermembrane supercurrent vector superfield

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- Now we see that the $\propto \delta H^a$ variation of the supermembrane action gives us the vector supercurrent of the form

$$
\begin{aligned}
J_a \;=\; & \int_{W^3} \frac{1}{2\hat{E}} \hat{E}^b \wedge \hat{E}^\alpha \wedge \hat{E}^\beta \, \sigma_{ab\alpha\beta} \delta^8(Z-\hat{Z}) - \\
& - \int_{W^3} \frac{i}{2\hat{E}} \left(*\hat{E}_a \wedge \hat{E}^\alpha - \frac{i}{2} \hat{E}^b \wedge \hat{E}^c \wedge \hat{\bar{E}}_{\dot\beta} \epsilon_{abcd} \tilde{\sigma}^{d\dot\beta\alpha} \right) \mathcal{D}_\alpha \delta^8(Z-\hat{Z}) + c.c + \\
& + \int_{W^3} \frac{1}{2\cdot 4! \hat{E}} \hat{E}^b \wedge \hat{E}^c \wedge \hat{E}^d \, \epsilon_{abcd} \left(\mathcal{D}\mathcal{D} - \frac{1}{2}\bar{R} \right) \delta^8(Z-\hat{Z}) + c.c. + \\
& + \int_{W^3} \frac{1}{4!\hat{E}} * \hat{E}_b \wedge \hat{E}^b \, G_a \, \delta^8(Z-\hat{Z}) - \\
& - \int_{W^3} \frac{1}{4!\hat{E}} * \hat{E}_c \wedge \hat{E}^b \tilde{\sigma}^{d\dot\alpha\alpha} \left(3\delta_a^c \delta_b^d - \delta_a^d \delta_b^c \right) [\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}] \delta^8(Z-\hat{Z}) \,,
\end{aligned}
$$

where $\hat{E} = sdet(E_M{}^A(\hat{Z}))$ and $\delta^8(Z) := \frac{1}{16}\,\delta^4(x)\,\theta\theta\,\bar\theta\bar\theta$ is the superspace delta function which obeys $\int d^8 Z\, \delta^8(Z-Z')f(Z) = f(Z')$ for any superfield $f(Z)$.

SUGRA superfield eqs and eqs. for supercurrent

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- The supercurrent enters the vector superfield eq. of SUGRA $G_a = J_a$ which follows from the action of the SUGRA+SM interacting system

$$S = S_{SG} + \frac{1}{6} S_{p=2} = \int d^8 Z E(Z) + \frac{1}{12} \int d^3\xi \sqrt{g} - \frac{1}{6} \int\limits_{W^3} \hat{C}_3 \; ,$$

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- where the *real* superfield $\mathcal{X} = \mathcal{X}^*$ is given by

$$\mathcal{X} = \frac{i}{E} \int_{W^3} \hat{E}^a \wedge \hat{E}^\alpha \wedge \hat{\bar{E}}^{\dot{\alpha}} \sigma^a_{\alpha\dot{\alpha}} \; \delta^8(Z - \hat{Z}) +$$

$$+ \int_{W^3} \left(- \frac{\hat{E}^b \wedge \hat{E}^a \wedge \hat{E}^\alpha}{4\hat{E}} \sigma_{ab\alpha}{}^\beta \mathcal{D}_\beta + c.c + \frac{\hat{E}^b \wedge \hat{E}^c \wedge \hat{E}^d}{2 \cdot 4! \hat{E}} \epsilon_{abcd} \tilde{\sigma}^{a\dot{\alpha}\alpha} [\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}] \right) \delta^8(Z - \hat{Z}) +$$

$$+ \int_{W^3} \left(i \frac{{}^*\hat{E}_a \wedge \hat{E}^a}{4! \hat{E}} (\mathcal{D}\mathcal{D} - \bar{R}) + c.c. + \frac{1}{4! \hat{E}} \hat{E}^b \wedge \hat{E}^c \wedge \hat{E}^d \epsilon_{abcd} G^a \right) \delta^8(Z - \hat{Z}) \, .$$

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- Notice that $\bar{\mathcal{D}}^{\dot\alpha} G_{\alpha\dot\alpha} = -\mathcal{D}_\alpha R$ and c.c. $\Rightarrow$

$$\bar{\mathcal{D}}^{\dot\alpha} J_{\alpha\dot\alpha} = i \mathcal{D}_\alpha \mathcal{X} \, , \qquad \mathcal{D}^\alpha J_{\alpha\dot\alpha} = -i \bar{\mathcal{D}}_{\dot\alpha} \mathcal{X} \, .$$

Outline

WZ$_{\hat{\theta}=0}$ gauge

- We use the general coordinate invariance to fix the Wess–Zumino (WZ) gauge on supergravity superfields

$$
\begin{aligned}
i_{\underline{\theta}} E^{\alpha} \quad &:= \quad \theta^{\underline{\breve{\alpha}}} E_{\underline{\breve{\alpha}}}{}^{\alpha} = \theta^{\alpha} \,, \qquad i_{\underline{\theta}} E^{\dot{\alpha}} := \theta^{\underline{\breve{\alpha}}} E_{\underline{\breve{\alpha}}}{}^{\dot{\alpha}} = \bar{\theta}^{\dot{\alpha}} \,, \\
& \qquad \theta^{\alpha} := \theta^{\underline{\breve{\beta}}} \delta_{\underline{\breve{\beta}}}^{\alpha} \,, \qquad \bar{\theta}^{\dot{\alpha}} := \theta^{\underline{\breve{\beta}}} \delta_{\underline{\breve{\beta}}}^{\dot{\alpha}} \,, \\
i_{\underline{\theta}} E^{\underline{a}} \quad &:= \quad \theta^{\underline{\breve{\alpha}}} E_{\underline{\breve{\alpha}}}{}^{\underline{a}} = 0 \,, \quad i_{\theta} w^{\underline{ab}} := \theta^{\underline{\breve{\beta}}} w_{\underline{\breve{\beta}}}^{\underline{ab}} = 0
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- and the local spacetime SUSY to set to zero the fermionic Goldstone field of the supermembrane,

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\hat{\theta}^{\underline{\alpha}}(\xi) = 0 \qquad \Leftrightarrow \qquad \hat{\theta}^{\alpha}(\xi) = 0\,, \qquad \hat{\bar{\theta}}^{\dot{\alpha}}(\xi) = 0\,.
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- The leading componet of supervielbein matrix has a triangular form

$$
E_N{}^A|_{\theta=0} = \begin{pmatrix} e_\nu^a(x) & \psi_\nu^{\underline{\alpha}}(x) \\ 0 & \delta_{\breve\beta}{}^{\underline{\alpha}} \end{pmatrix} \qquad \Rightarrow \qquad E_A{}^N|_{\theta=0} = \begin{pmatrix} e_a^\nu(x) & -\psi_a^{\breve\beta}(x) \\ 0 & \delta_{\underline{\alpha}}{}^{\breve\beta} \end{pmatrix}
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- Relation between the leading componet of T_{ab}^α and the true gravitino field strength

$$T_{ab}{}^\alpha|_{\theta=0} = 2e_a^\mu e_b^\nu \mathcal{D}_{[\mu}\psi_{\nu]}^\alpha(x) - \frac{i}{4}(\psi_{[a}\sigma_{b]})_{\dot\beta} G^{\alpha\dot\beta}|_{\theta=0} - \frac{i}{4}(\bar\psi_{[a}\tilde\sigma_{b]})^\alpha R|_{\theta=0}$$

Current superfields in the WZ$_{\hat{\theta}=0}$ gauge

- We find that the vector and scalar superfields have the following form,

$$J_{\alpha\dot{\alpha}}|_{\hat{\theta}=0} = \frac{\theta_\beta\,\bar{\theta}_{\dot{\beta}}}{8}\left(3\mathcal{P}_a{}^b(x)\sigma^a_{\alpha\dot{\alpha}}\tilde{\sigma}_b^{\beta\dot{\beta}} - 2\delta_\alpha{}^\beta\delta_{\dot{\alpha}}{}^{\dot{\beta}}\mathcal{P}_b{}^b(x)\right)$$
$$-i\frac{(\theta\theta-\bar{\theta}\bar{\theta})}{32}\sigma^a_{\alpha\dot{\alpha}}\mathcal{P}_a(x) + \propto \underline{\theta}^{\wedge 3}$$
$$\mathcal{X}|_{\hat{\theta}=0} = -\frac{\theta\sigma^a\bar{\theta}}{16}\mathcal{P}_a + i\frac{(\theta\theta-\bar{\theta}\bar{\theta})}{16}\mathcal{P}_a{}^a(x) + \propto \underline{\theta}^{\wedge 3}$$

Current superfields in the $\mathrm{WZ}_{\hat\theta=0}$ gauge

- We find that the vector and scalar superfields have the following form,

$$J_{\alpha\dot\alpha}|_{\hat\theta=0} = \frac{\theta_\beta\,\bar\theta_{\dot\beta}}{8}\left(\,3\mathcal{P}_a{}^b(x)\sigma^a_{\alpha\dot\alpha}\tilde\sigma_b^{\beta\dot\beta} - 2\delta_\alpha{}^\beta\delta_{\dot\alpha}{}^{\dot\beta}\mathcal{P}_b{}^b(x)\right)$$
$$-i\frac{(\theta\theta-\bar\theta\bar\theta)}{32}\sigma^a_{\alpha\dot\alpha}\mathcal{P}_a(x) + \propto \underline{\theta}^{\wedge 3}$$
$$\mathcal{X}|_{\hat\theta=0} = -\frac{\theta\sigma^a\bar\theta}{16}\mathcal{P}_a + i\frac{(\theta\theta-\bar\theta\bar\theta)}{16}\mathcal{P}_a{}^a(x) + \propto \underline{\theta}^{\wedge 3}$$

- Where we have introduced the current pre-potential fields,

$$\mathcal{P}_a{}^b(x) \;:=\; \int_{W^3} \frac{1}{\hat e} * \hat e_a \wedge \hat e^b\, \delta^4(x-\hat x)\ ,$$
$$\mathcal{P}_a(x) \;:=\; \int_{W^3} \frac{1}{\hat e}\epsilon_{abcd}\hat e^b \wedge \hat e^c \wedge \hat e^d\;\; \delta^4(x-\hat x) =$$
$$= \; e^\mu_a(x)\int_{W^3}\epsilon_{\mu\nu\rho\sigma}d\hat x^\nu \wedge d\hat x^\rho \wedge d\hat x^\sigma\;\; \delta^4(x-\hat x)$$

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- There is no explicit supermembrane contributions to the Rarita-Schwinger equations ,

Einstein equation in the WZ$_{\hat{\theta}=0}$ gauge

- We calculate the Einstein eq. using the vector and scalar current superfields,

$$R_{bc}{}^{ac}|_{\theta=0\ ,\ \hat{\theta}=0} \quad = \quad -\frac{3}{32}\,T_2\left(\mathcal{P}_b{}^a(x) - \frac{1}{2}\delta^a_b\mathcal{P}_c{}^c(x)\right) + \frac{3}{64}(R+\bar{R})^2|_{\theta=0}\delta^a_b\ .$$

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$$R - \bar{R} = -iT_2\mathcal{X} \qquad \Rightarrow \qquad \partial_\mu(R+\bar{R})|_{\theta=0} = \frac{T_2}{16}\int_{W^3} \epsilon_{\mu\nu\rho\sigma}d\hat{x}^\nu \wedge d\hat{x}^\rho \wedge d\hat{x}^\sigma\ \delta^4(x-\hat{x})$$

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- The solution can be written in the form,

$$R(x) + \bar{R}(x) = 8c + \frac{T_2}{16}\int_{x_0}^{x} d\tilde{x}^\mu \int_{W^3} \epsilon_{\mu\nu\rho\sigma} d\hat{x}^\nu \wedge d\hat{x}^\rho \wedge d\hat{x}^\sigma\ \delta^4(\tilde{x}-\hat{x})$$

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$$\Theta(x, x_0|\hat{x}) := \int_{x_0}^{x} d\tilde{x}^{\mu} \int_{W^3} \epsilon_{\mu\nu\rho\sigma} d\hat{x}^{\nu} \wedge d\hat{x}^{\rho} \wedge d\hat{x}^{\sigma} \; \delta^4(\tilde{x} - \hat{x})$$

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- And finally we obtain,

$$R_{bc}{}^{ac}(x) = -\frac{3T_2}{32}\left(\mathcal{P}_b{}^a(x) - \frac{1}{2}\delta_b^a \mathcal{P}_c{}^c(x)\right)$$
$$+3\delta_b^a\left(c^2 + \left(\left(\frac{T_2}{128} + c\right)^2 - c^2\right)\Theta(x, x_0|\hat{x})\right)$$

Supermembrane contributions to Einstein equation

- We can separate these contributions in three different classes

$$R_{acb}{}^{c}(x) = \eta_{ab}\, 3c^2 + T_2 \left(\mathcal{T}_{ab}^{sing}(x) + \mathcal{T}_{ab}^{reg}(x) \right)$$

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- The last one contains regular terms proportional to the supermembrane tension,

$$\mathcal{T}^{reg}_{ab}(x) = \eta_{ab}\mathcal{T}^{reg}(x)\;, \qquad \mathcal{T}^{reg}(x) = +\frac{3T_2}{64}\left(\frac{T_2}{256} + c \right) \Theta(x, x_0|\hat{x})\;.$$

Regular supermembrane contributions

- Considering the Einstein eq. in two pieces of space time separated by the supermembrane worldvolume

$$
\begin{aligned}
M^4_+ \quad : \qquad & R_{acb}{}^c(x) = 3\eta_{ab} \left(\frac{T_2}{128} + c \right)^2 \\
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- These values coincide if $c = -\frac{T_2}{256}$, but we do not find any reason for such choice

Supersymmetric solutions of the interacting system eqs.

- If we search purely bosonic supersymmetric solutions ($\psi_\mu^\alpha = 0$), we study Killing spinor equations ($\delta_\epsilon \psi_\mu^\alpha = 0$)

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Integrability conditions

- Applying the covariant derivative and using the Ricci identities $DD\epsilon^{\alpha} = -\frac{1}{4} R^{ab} \epsilon^{\beta} \sigma_{ab\,\beta}{}^{\alpha}$

$$M^4_- \quad : \qquad R^{ab} \epsilon^{\beta} \sigma_{ab\,\beta}{}^{\alpha} = \frac{1}{4} |c|^2 e^d \wedge e^c \, \epsilon^{\beta} \sigma_{cd\,\beta}{}^{\alpha} \; ,$$

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- These equations solve our equations of motion and describe the completely SUSY solution (At least modulo singular terms)

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