Non-linear deformations of duality-symmetric theories

DMITRI SOROKIN

Dipartimento di Fisica e Astronomia "Galileo Galilei"
Università degli Studi di Padova
and
INFN, Sezione di Padova

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Motivation

- Duality symmetry plays important role in many theor. models of physical interest
- N=8 supergravity is invariant under $\boldsymbol{E_{7(7)}}$ *(Cremmer & Julia '79)*

$$\mathcal{F}'^{i}_{\mu\nu} = (F^{A}_{\mu\nu}, G^{\bar{A}}_{\mu\nu}) \quad i = 1,\dots,56 \ \text{ of } \ E_{7(7)} \ \text{ and } \ A, \bar{A} = 1,\dots,28 \ \text{ of } \ SU(8)$$

electric magnetic

On-shell linear (twisted self-) duality:

$$G^{\bar{A}}_{\mu\nu} = \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma A} \quad \Rightarrow \quad \mathcal{F}'^{-i}_{\mu\nu} \equiv \mathcal{F}'^{i}_{\mu\nu} - \tfrac{1}{2}\Omega^{i}{}_{j}\varepsilon_{\mu\nu\rho\sigma}\mathcal{F}'^{\rho\sigma j} = 0, \quad \Omega^{i}{}_{k}\Omega^{k}{}_{j} = -\delta^{i}_{j}$$

- N=8 supergravity is perturbatively finite at 3 and 4 loops *(Bern et. al.)*
- Assumption: SUSY + $\boldsymbol{E_{7(7)}}$ may be in charge of the absence of divergences *(Kallosh)*
- $\boldsymbol{E_{7(7)}}$-invariant counterterms can appear at 7 loops $\partial^{2k}F^4, \partial^{2k}R^4$

Motivation

- higher-order deformations $\partial^{2k}F^4$ in the effective action will lead to a non-linear deformation of the twisted self-duality condition

$$L = \tfrac{1}{4}F^2 + \partial^{2k}F^4 + \cdots,$$

$$\tilde{G} = 2\frac{\delta L(F)}{\delta F} \quad \Rightarrow \quad \mathcal{F}'^{\,i}_{-} = \frac{\delta \Delta(\mathcal{F}')}{\delta \mathcal{F}'^{\,i}_{+}} \neq 0, \qquad \mathcal{F}'^{\,i}_{+} = \mathcal{F}'^{\,i} + \Omega^{i}{}_{j}\tilde{\mathcal{F}}'^{\,i}, \quad \tilde{G}_{\mu\nu} \equiv \tfrac{1}{2}\varepsilon_{\mu\nu\rho\lambda}G^{\rho\lambda}$$

$$\mathcal{F}'^{\,i} = (F^{A}, G^{\bar{A}})$$

$\Delta(\mathcal{F})$ - duality-invariant counterterm

- Questions to answer:
  - how, exactly, possible higher-order terms may deform the effective action and duality relation between ‘electric’ and ‘magnetic’ fields, while keeping duality symmetry?
  - check whether this deformation is compatible with supersymmetry

- in this talk we shall mainly concern with the first problem
- brief comments on supersymmetry in conclusion

Two ways of dealing with duality-symmetric theories

I. Lagrangian depends only on 'electric' fields *L(F)* and is not duality-invariant

- **Duality symmetry** manifests itself **only on-shell**: $\tilde{G} = 2\frac{\delta L(F)}{\delta F} = F + \Delta(F)$

  in the linear case

$$\mathcal{F}'^{i} = (F, G), \qquad \delta\mathcal{F}'^{i} = M^{i}{}_{j}\mathcal{F}'^{j} \text{ - linear duality transform } \quad M^{i}{}_{j} \subset Sp(2N)$$

- The variation of *L(F)* under duality transform should satisfy a condition *(Gaillard-Zumino '81, '97; Gibbons-Rasheed '95)*

$$\delta L = \tfrac{1}{4}\delta(F\tilde{G}) \quad \Rightarrow \quad F\tilde{F} + G\tilde{G} = 0$$

II. Lagrangian depends on both 'electric' and 'magnetic' fields $L(\mathcal{F}'^{i})$.

It is manifestly duality invariant. Duality condition follows from e.o.m.

Subtleties with space-time covariance

Duality-invariant actions

- Space-time invariance is not manifest

  (Zwanziger '71, Deser & Teitelboim '76, Henneaux & Teitelboim '87, ...)

Example: duality-symmetric Maxwell action for $\mathcal{F}^{i} = dA^{i} \quad (i=1,2)$

$$L = \tfrac{1}{8}\mathcal{F}^{i}_{\mu\nu}\mathcal{F}^{i\mu\nu} + \tfrac{1}{4}(\mathcal{F}^{i}_{0a} - \varepsilon^{ij}\tilde{\mathcal{F}}^{j}_{0a})(\mathcal{F}^{0ai} - \varepsilon^{ij}\tilde{\mathcal{F}}^{0aj}) \qquad \mu = (0,a) \quad a = 1,2,3$$

breaks manifest Lorentz invariance

Modified Lorentz invariance: $\delta A^{i}_{\mu} = \delta_{\Lambda} A^{i}_{\mu} + x^{a}\Lambda_{a}{}^{0}(\mathcal{F}^{i}_{0\mu} - \varepsilon^{ij}\tilde{\mathcal{F}}^{j}_{0\mu})$

Twisted self-duality condition is obtained by integrating the e.o.m. :

$$\frac{\delta L}{\delta A^{i}} = 0 \;\Rightarrow\; \mathcal{F}^{i}_{\mu\nu} - \varepsilon^{ij}\tilde{\mathcal{F}}^{j}_{\mu\nu} = 0 \;\Rightarrow\; F^{1}_{\mu\nu} = \tilde{F}^{2}_{\mu\nu}$$

Space-time covariant and duality-invariant action

- Space-time covariance can be restored by introducing an auxiliary scalar field $a(x)$ *(Pasti, D.S. & Tonin '95)*

$$L_{nonc} = \tfrac{1}{8} \mathcal{F}'^{i}_{\mu\nu} \mathcal{F}'^{i\mu\nu} + \tfrac{1}{4} (\mathcal{F}'^{i}_{0a} - \varepsilon^{ij} \tilde{\mathcal{F}}'^{j}_{0a})(\mathcal{F}'^{0ai} - \varepsilon^{ij} \tilde{\mathcal{F}}'^{0aj}) \quad \mu = (0, a) \quad a = 1,2,3$$

$$L_{cov} = \tfrac{1}{8} \mathcal{F}'^{i}_{\mu\nu} \mathcal{F}'^{i\mu\nu} + \tfrac{1}{4} \upsilon^{\mu} (\mathcal{F}'^{i}_{\mu\nu} - \varepsilon^{ij} \tilde{\mathcal{F}}'^{j}_{\mu\nu})(\mathcal{F}'^{\nu\lambda i} - \varepsilon^{ij} \tilde{\mathcal{F}}'^{\nu\lambda j}) \upsilon_{\lambda}(x)$$

$$\upsilon_{\mu}(x) = \frac{\partial_{\mu} a(x)}{\sqrt{(\partial a)^2}}, \qquad \upsilon_{\mu} \upsilon^{\mu} = 1$$

Local symmetries:

$$\delta A^{i}_{\mu} = \partial_{\mu} \lambda^{i}(x)$$

$$\delta_{I} A^{i}_{\mu} = \Phi(x) \partial_{\mu} a(x), \qquad \delta_{I} a(x) = 0 \quad \longrightarrow \quad \upsilon^{\mu} A^{i}_{\mu} \text{ - is pure gauge}$$

$$\delta_{II} a(x) = \varphi(x), \qquad \delta_{II} A^{i}_{\mu} = \frac{\varphi(x)}{\sqrt{(\partial a)^2}} \upsilon^{\nu} (\mathcal{F}'^{i}_{\mu\nu} - \varepsilon^{ij} \tilde{\mathcal{F}}'^{j}_{\mu\nu}) \quad \longrightarrow \quad \text{gauge fixing}$$

$$a(x) = x^{0}, \quad \upsilon_{\mu} = \delta^{0}_{\mu}$$

Non-linear generalization

Another form of the Lagrangian:

$$L_{\text{cov}} = \tfrac{1}{4}\Omega^{ij}(\upsilon^{\mu}\mathcal{F}'^{i}_{\mu\nu})\,(\upsilon_{\lambda}\widetilde{\mathcal{F}'}^{\lambda\nu j}) - \tfrac{1}{4}(\upsilon^{\mu}\widetilde{\mathcal{F}'}^{i}_{\mu\nu})(\upsilon_{\lambda}\widetilde{\mathcal{F}'}^{\lambda\nu i}), \quad \Omega^{2} = -1, \quad i, j = 1,\ldots,2N$$

pure gauge component $\upsilon^{\mu}A^{i}_{\mu}$ enters only the 1st term under the total derivative

$$\upsilon^{\mu}\widetilde{\mathcal{F}'}^{i}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\lambda}\upsilon^{\mu}\mathcal{F}'^{\rho\lambda i}$$ does not contain $\upsilon^{\mu}A^{i}_{\mu}$

Higher-order Lagrangian:

$$L = \tfrac{1}{4}\Omega^{ij}(\upsilon^{\mu}\mathcal{F}'^{i}_{\mu\nu})\,(\upsilon_{\lambda}\widetilde{\mathcal{F}'}^{\lambda\nu j}) - \tfrac{1}{4}(\upsilon^{\mu}\widetilde{\mathcal{F}'}^{i}_{\mu\nu})(\upsilon_{\lambda}\widetilde{\mathcal{F}'}^{\lambda\nu i}) - \tfrac{1}{4}\mathcal{L}[i_{\upsilon}\widetilde{\mathcal{F}'}, di_{\upsilon}\widetilde{\mathcal{F}'}, \phi]$$

where $(i_{\upsilon}\widetilde{\mathcal{F}'})_{\nu} = \upsilon^{\mu}\widetilde{\mathcal{F}'}_{\mu\nu}$

By construction L is invariant under $\delta_{I}A^{i}_{\mu} = \Phi(x)\partial_{\mu}a(x), \quad \delta_{I}a(x) = 0$

Non-linear Lagrangian & local *a(x)*-symmetry

$$L=\tfrac{1}{4}\Omega^{ij}(\upsilon^{\mu}\mathcal{F}'^{i}_{\mu\nu})(\upsilon_{\lambda}\tilde{\mathcal{F}}'^{\lambda\nu j})-\tfrac{1}{4}(\upsilon^{\mu}\tilde{\mathcal{F}}'^{i}_{\mu\nu})(\upsilon_{\lambda}\tilde{\mathcal{F}}'^{\lambda\nu i})-\tfrac{1}{4}\mathcal{L}[i_{\upsilon}\tilde{\mathcal{F}}', di_{\upsilon}\tilde{\mathcal{F}}',\ldots]$$

2nd local symmetry in the linear case:

$$\delta_{II}a(x)=\varphi(x),\qquad \delta_{II}A^{i}_{\mu}=\frac{\varphi(x)}{\sqrt{(\partial a)^2}}\upsilon^{\nu}(\mathcal{F}'^{i}_{\mu\nu}-\Omega^{ij}\tilde{\mathcal{F}}'^{j}_{\mu\nu})\ =0\ \text{ on shell}$$

A^{i} equation of motion:

$$\frac{\delta L}{\delta A^{i}}=d\left(\upsilon(i_{\upsilon}\mathcal{F}'^{i}-\Omega^{ij}i_{\upsilon}\tilde{\mathcal{F}}'^{j}-\Omega^{ij}\frac{\delta\mathcal{L}}{\delta(i_{\upsilon}\tilde{\mathcal{F}}'_{j})})\right)=0\ \Rightarrow\ \boxed{\upsilon^{\nu}(\mathcal{F}'^{i}_{\mu\nu}-\Omega^{ij}\tilde{\mathcal{F}}'^{j}_{\mu\nu})-\Omega^{ij}\frac{\delta\mathcal{L}}{\delta(\upsilon_{\nu}\tilde{\mathcal{F}}'^{\mu\nu}_{j})}=0}$$

2nd local symmetry in non-linear case:

$$\delta_{II}a(x)=\varphi(x),\qquad \delta_{II}A^{i}_{\mu}=\frac{\varphi(x)}{\sqrt{(\partial a)^2}}\left(\upsilon^{\nu}(\mathcal{F}'^{i}_{\mu\nu}-\Omega^{ij}\tilde{\mathcal{F}}'^{j}_{\mu\nu})-\Omega^{ij}\frac{\delta\mathcal{L}}{\delta(\upsilon_{\nu}\tilde{\mathcal{F}}'^{\mu\nu j})}\right)$$

Consistency condition on non-linear deformation $\mathcal{L}(\mathcal{F})$

$$\delta_{II} L = 0 \quad \Rightarrow \quad \Omega^{ij} d \left[\frac{v}{\sqrt{(\partial a)^2}} \left(i_v \tilde{\mathcal{F}}^i + \frac{\delta \mathcal{L}}{2\delta(i_v \tilde{\mathcal{F}}^i)} \right) \frac{\delta \mathcal{L}}{\delta(i_v \tilde{\mathcal{F}}^j)} \right] = 0$$

The condition on $\mathcal{L}$ ensures the auxiliary nature of the scalar $a(x)$

upon gauge fixing $a(x)$ it ensures non-manifest space-time invariance

Known examples:

- Born-Infeld-like form of the M5-brane action
  (Perry & Schwarz '96; Pasti, D.S. and Tonin '97)
- Born-Infeld-like form of the duality-symmetric D3-brane action
  (Berman '97; Nurmagambetov '98)
- New Born-Infeld-like deformations *(Kuzenko et. al, Bossard & Nicolai; Kallosh et. al '11)*

$a(x)$-independence of twisted self-duality condition

$$\upsilon^{\nu}(\mathcal{F}'^{\,i}_{\mu\nu} - \Omega^{ij}\tilde{\mathcal{F}}'^{\,j}_{\mu\nu}) - \Omega^{ij}\frac{\delta\mathcal{L}}{\delta(\upsilon_{\nu}\tilde{\mathcal{F}}'^{\,\mu\nu}_{j})} = 0$$

$\Downarrow$

$$\mathcal{F}'^{\,i} - \Omega^{ij} * \mathcal{F}'^{\,j} = \upsilon\frac{\delta\mathcal{L}}{\delta(i_{\upsilon}\tilde{\mathcal{F}}'_{i})} - \Omega^{ij} * \upsilon\frac{\delta\mathcal{L}}{\delta(i_{\upsilon}\tilde{\mathcal{F}}'_{j})} = \frac{\delta\Delta(\mathcal{F}')}{\delta\mathcal{F}'^{\,i}_{+}}, \qquad \mathcal{F}'^{\,i}_{+} = \mathcal{F}'^{\,i} + \Omega^{ij} * \mathcal{F}'^{\,j}$$

should not depend on $\upsilon(x) \sim da(x)$
independently of gauge fixing

This establishes on-shell relation between manifestly duality-symmetric and Gaillard-Zumino approach to the construction of non-linear self-dual theories

Main issues: Whether counterterms of N=8,4 sugra can provide the form of $\Delta(\mathcal{F})$?
If yes, whether this deformation is consistent with supersymmetry ?

Supersymmetry issue

- Counterterms $\partial^{2k}F^4$ that can appear at 7 loops in *N*=8 sugra are supersymmetric and $\boldsymbol{E_{7(7)}}$- invariant on the mass-shell, i.e.

modulo linear twisted self-duality $\quad \mathcal{F}_-^{\prime i} = \mathcal{F}^{\prime i} - \Omega^i{}_j \widetilde{\mathcal{F}}^{\prime i} = 0$

$$\Longrightarrow \quad \Delta_0(\mathcal{F}^{\prime i}, \phi) = \Delta_0(\mathcal{F}_+^{\prime i}, \phi) = \Delta_0(\mathrm{F}^A, \phi)$$

- When included into the effective action, $\mathcal{I}_0(\mathcal{F})$ deforms duality condition

$$\mathcal{F}_-^{\prime i} = \frac{\delta\Delta_0(\mathcal{F}^\prime, \phi)}{\delta\mathcal{F}_+^{\prime i}} \neq 0 \quad \Longrightarrow \quad \Delta(\mathcal{F}_+^{\prime i}, \mathcal{F}_-^{\prime i}, \phi), \quad \Delta(\mathcal{F}_+^{\prime i}, \mathcal{F}_-^{\prime i}, \phi)\Big|_{\mathcal{F}_-^{\prime i}=0} = \Delta_0$$

whose form is determined by GZ- *Gibbons-Rasheed* condition or by space-time invariance of the deformed action

Supersymmetry of $\Delta(\mathcal{F}_+^{\prime i}, \mathcal{F}_-^{\prime i}, \phi)$ should be checked

Standard *N*=8 superspace methods are **not applicable.** Use **component formalism**

Supersymmetry of duality-symmetric actions

- Example: duality-symmetric N=1 Maxwell action $\mathcal{F}'^i = dA^i \quad (i=1,2)$

$$L_{N=1} = \tfrac{1}{8}\mathcal{F}'^i_{\mu\nu}\mathcal{F}'^{i\mu\nu} + \tfrac{1}{4}\upsilon^\mu(\mathcal{F}'^i_{\mu\nu} - \varepsilon^{ij}\widetilde{\mathcal{F}}'^j_{\mu\nu})(\mathcal{F}'^{\nu\lambda i} - \varepsilon^{ij}\widetilde{\mathcal{F}}'^{\nu\lambda j})\upsilon_\lambda + \tfrac{i}{2}\overline{\psi}\gamma^\mu\partial_\mu\psi$$

Susy transformations *(Schwarz & Sen '93, Pasti, D.S. & Tonin '95)*

$$\delta A^i_\mu = i\overline{\psi}\gamma_\mu\varsigma^i, \qquad \varsigma^i = \varepsilon^{ij}\gamma_5\varsigma^j, \qquad \delta_\varsigma a(x) = 0$$

$$\delta\psi = \tfrac{1}{8}K^i_{\mu\nu}\gamma^{\mu\nu}\varsigma^i, \qquad K^i_{\mu\nu} = \mathcal{F}'^i_{\mu\nu} + \upsilon_{[\mu}(\mathcal{F}'^i_{\nu]\rho} - \varepsilon^{ij}\widetilde{\mathcal{F}}'^i_{\nu]\rho})\upsilon^\rho$$

On shell $(F^2 = -\tilde{F}^1)$: $\quad \delta A^1_\mu = i\overline{\psi}\gamma_\mu\varsigma^1, \quad \delta\psi = \tfrac{1}{4}F^1_{\mu\nu}\gamma^{\mu\nu}\varsigma^1$

- In the non-linear case: $K^i = \mathcal{F}'^i + \upsilon(i_\upsilon\mathcal{F}'^i_- - \dfrac{\delta\mathcal{L}(\mathcal{F},\psi)}{\delta\mathcal{F}'^i_+})$

Non-linear duality, supersymmetry and UV

Examples:

- N=1,2,3,4, D=4 Born-Infeld theories (D3-branes) *(known since '95)*
- Abelian N=(2,0) D=6 self-dual theory on the worldvolume of the M5-brane ('96)
- BI models (including higher-order derivatives) coupled to N=1,2 D=4 sugra

  (Kuzenko and McCarthy '02, Kuzenko '12, Kallosh et. all '12…)

In most of the known examples non-linear deformation of duality is related to a partial spontaneous breaking of supersymmetry

Issues:

- Whether non-linear deformations are possible for vector fields inside supergravity multiplets, in particular, in N=4,8 supergravities?

  (for N=2 sugra, *Kallosh et.all 08.2012)*
- Whether this interplay between dualities and supersymmetry may shed light on the UV behavior of N=4,8 supergravities?