ON THE T-DUALITY SYMMETRIC CLOSED BOSONIC STRING THEORY

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PLAN OF THE TALK

- Motivation.
- T-duality in bosonic closed string theory.
- Looking for a T-duality symmetric formulation:
  - circle compactification;
  - toroidal compactification.
- Implications on the gravitational effective theory.

References

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MOTIVATION

The presence of compact dimensions implies the existence of different kind of modes for bosonic closed strings:

momentum modes $\Rightarrow$ $p_a = \dfrac{k_a}{R}$, $k_a \in Z$ quantized along the compact direction.

winding modes $\Rightarrow$ $X^a(\tau,\sigma+\pi) = X^a(\tau,\sigma) + 2\pi R w^a \; ; \; w^a \in Z$

dual coordinates

$$\begin{cases} X^a \; , \; p_a \\ \tilde{X}_a \; , \; w^a \end{cases}$$

representing the number of times the string winds around the compact dimension

WHAT PROVIDES THIS “NEW” COORDINATE?
T-DUALITY

T-duality is an old subject in String Theory. It implies that in many cases two different geometries for the extra-dimensions are physically equivalent. In the simplest case, a circle of radius *R* is equivalent to a circle of radius *α'/R.*

T-duality is a symmetry for the bosonic closed string theory and, in the case of a circle compactification, it is encoded by the following transformations:

$$R \leftrightarrow \frac{\alpha'}{R} \qquad ; \qquad k \leftrightarrow w$$

which imply a transformation on the string coordinate *X* along the compact dimension:

$$X(\tau,\sigma) = X_L(\tau+\sigma) + X_R(\tau-\sigma) \leftrightarrow \tilde{X}(\tau,\sigma) = X_L(\tau+\sigma) - X_R(\tau-\sigma)$$

$$X \leftrightarrow \tilde{X}$$

associated with the winding mode

IT WOULD BE INTERESTING TO FIND A MANIFESTLY T-DUAL INVARIANT FORMULATION OFTHE BOSONIC CLOSED STRING THEORY!

All of this implies that, if interested in writing down the complete effective field theory of a compactified bosonic closed string, one has to include both *momentum* excitations and *winding* excitations or, equivalently X^a *and* $\tilde{X}_a$.

The fields associated with the string states will depend on $X^i = (X^a, \tilde{X}_a, X^\mu)$.

The effective closed string field theory would look like:

$$S = \int dX^a d\tilde{X}_a \, dX^\mu \, L\left(X^a, \tilde{X}_a, X^\mu\right).$$

Hence, it is a fact that the closed string effective field theory is a DOUBLE FIELD THEORY.

This has to be true, in particular, for the well-known effective gravitational action involving the fields associated with string massless states $g_{\mu\nu}$, $B_{\mu\nu}$ and φ:

$$S = \int dX^{\mu} \sqrt{|g|}\, e^{-2\varphi} \left[R + 4(\partial\varphi)^2 - \frac{1}{12} H^2 \right]$$

QUESTIONS

WHAT DOES THIS ACTION BECOME IN THE LIGHT THAT ALL THE FIELDS ARE "DOUBLED"?
WHAT SYMMETRIES AND WHAT PROPERTIES WOULD IT HAVE?

deriving from the question:

HOW WOULD THE CLOSED STRING LOOK LIKE WHEN T- DUALITY IS MADE MANIFEST?

…hopefully shedding light on aspects of string gravity so far unexplored.

T-DUALITY IN BOSONIC CLOSED STRING THEORY

Circle compactification

World-sheet action for a bosonic closed string in a Minkowski space background:

$$S = -\frac{T}{2}\int_0^{\pi} d\sigma \int_{-\infty}^{+\infty} d\tau\, \eta^{\alpha\beta}\partial_\alpha X^i \partial_\beta X^j \eta_{ij}$$

$$\Rightarrow \quad S = \frac{T}{2}\int_0^{\pi} d\sigma \int_{-\infty}^{+\infty} d\tau\, (\dot{X}^2 - X'^2) \qquad \dot{X} = \partial_\tau X \;\; ; \;\; X' = \partial_\sigma X$$

Single coordinate compactified on a circle of radius *R*.

Compactification is defined by the period identification: $X \approx X + 2\pi R w \qquad w \in Z$

$$(\partial_\tau^2 - \partial_\sigma^2)X = 0 \quad \Rightarrow \quad X(\tau,\sigma) = X_L(\tau+\sigma) + X_R(\tau-\sigma)$$

$$X(\tau,\sigma) = x + 2\alpha' p\tau + 2Rw\sigma + \frac{i}{2}\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\bar{\alpha}_n e^{-2in(\tau+\sigma)} + \frac{i}{2}\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n e^{-2in(\tau-\sigma)}$$

$$p = \frac{k}{R} \qquad k \in Z$$

The momentum in the compact direction is quantized

X_L and X_R describe, respectively, “left-moving” and “right-moving” modes:

$$X_L(\tau+\sigma)=\overset{x_L}{\frac{1}{2}(x+\tilde{x})}+\overset{\alpha' p_L}{\left(\alpha'\frac{k}{R}+wR\right)}(\tau+\sigma)+\frac{i}{2}\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\bar{\alpha}_n e^{-2in(\tau+\sigma)}\ ;$$

$$X_R(\tau-\sigma)=\underset{x_R}{\frac{1}{2}(x-\tilde{x})}+\underset{\alpha' p_R}{\left(\alpha'\frac{k}{R}-wR\right)}(\tau-\sigma)+\frac{i}{2}\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n e^{-2in(\tau-\sigma)}\ .$$

The Hamiltonian reads:

$$H=\frac{1}{2}p_L^2+\frac{1}{2}p_R^2+\sum_{n\neq 0}\alpha_{-n}\alpha_n+\sum_{n\neq 0}\bar{\alpha}_{-n}\bar{\alpha}_n=\frac{1}{2}p_L^2+\frac{1}{2}p_R^2+N+\bar{N}$$

It turns to be invariant under the transformations: T-DUALITY

under which:

$$R\leftrightarrow\tilde{R}=\frac{\alpha'}{R}\qquad;\qquad k\leftrightarrow w$$

$$p_L\to p_L\ ;\ p_R\to -p_R$$

$$\alpha_n\to\alpha_{-n}$$

$$\Longrightarrow$$

$$X_L\to X_L\ ;\ X_R\to -X_R$$

$$X(\tau,\sigma)\to X_L(\tau+\sigma)-X_R(\tau-\sigma)\equiv\tilde{X}(\tau,\sigma)$$

T-duality symmetry is a clear indication that ordinary geometric concepts can break down in string theory at the string scale.

The interchange of *w* and *k* means that the momentum excitations in one description correspond to winding mode excitations in the dual description and *viceversa.*

The T dual field is given by:

$$\tilde{X}(\tau,\sigma) = \tilde{x} + 2Rw\tau + 2\alpha'\frac{k}{R}\sigma + \frac{i}{2}\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\bar{\alpha}_n e^{-2in(\tau+\sigma)} + \frac{i}{2}\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n e^{-2in(\tau-\sigma)}$$

$$\partial_\sigma X = \partial_\tau \tilde{X} \quad ; \quad \partial_\tau X = \partial_\sigma \tilde{X}$$

duality conditions

$$\partial_\alpha \tilde{X} = -\varepsilon_\alpha^{\ \beta}\partial_\beta X \quad ; \qquad \varepsilon^{01} = -\varepsilon^{10} = 1$$

$$*dX = d\tilde{X} \qquad\qquad *dX_L = dX_L \quad , \quad *\,dX_R = -dX_R$$

dual

anti-dual

Looking for a T-duality symmetric formulation: circle compactification

A general scalar field theory in 2D Minkowski space theory with the usual Lagrangian density

$$L = -\frac{1}{2}\partial_\alpha \phi \, \partial^\alpha \phi = \frac{1}{2}(\dot{\phi}^2 - \phi'^2) \qquad \eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

admits a dual description $\quad L \to \frac{1}{2}(\dot{\tilde{\phi}}^2 - \tilde{\phi}'^2)$

with $\quad \partial_\sigma \phi = \partial_\tau \tilde{\phi} \ , \ \partial_\tau \phi = \partial_\sigma \tilde{\phi} \quad \longrightarrow \quad *d\phi = d\tilde{\phi}$

This can be easily seen, by noting that the two Lagrangians are equivalent to:

$$L = -\frac{1}{2}V^\alpha V_\alpha - \varepsilon^{\alpha\beta}\phi \, \partial_\beta V_\alpha$$

In fact, starting from

$$S = -\int d\tau\, d\sigma \left[-\frac{1}{2} V^{\alpha} V_{\alpha} - \varepsilon^{\alpha\beta} \phi\, \partial_{\beta} V_{\alpha} \right]$$

$$\delta_{\phi} S = 0 \Rightarrow \varepsilon^{\alpha\beta} \partial_{\beta} V_{\alpha} = 0$$

$$V_{\alpha} = \partial_{\alpha} \tilde{\phi}$$

arbitrary function

$$S = \int d\tau\, d\sigma \left[-\frac{1}{2} \partial_{\alpha} \tilde{\phi}\, \partial^{\alpha} \tilde{\phi} \right]$$

Alternatively:

$$\delta_{V^{\alpha}} S = 0 \Rightarrow V^{\alpha} = -\varepsilon^{\alpha\beta} \partial_{\beta} \phi$$

$$S = \int d\tau\, d\sigma \left[-\frac{1}{2} \partial_{\alpha} \phi\, \partial^{\alpha} \phi \right]$$

$$\int d\sigma \left[\phi\, \partial_{\tau} \phi \right] \Big|_{\tau=-\infty}^{\tau=+\infty} = \int d\tau \left[\phi\, \partial_{\sigma} \phi \right] \Big|_{\sigma=\sigma_0}^{\sigma=\sigma_1} = 0$$

$$\partial_{\alpha} \tilde{\phi} = -\varepsilon_{\alpha}^{\ \beta} \partial_{\beta} \phi$$

Duality conditions

It is possible to rewrite L in such a way that the two fields appear on an equal footing.

Procedure

- Regard p as a Lagrange multiplier:

$$L = p\dot{\phi} - \frac{1}{2}p^2 - \frac{1}{2}\phi'^2 \qquad\qquad p = \dot{\phi}$$

- and then add a second multiplier b for the constraint: $p = \tilde{\phi}'$

$$L' = \dot{\phi}\,\tilde{\phi}' - \frac{1}{2}\phi'^2 - \frac{1}{2}\tilde{\phi}'^2$$

[F. Bastianelli and P. van Nieuwenhuizen (1989)]

equations of motion

$$\partial_\sigma \phi = \partial_\tau \tilde{\phi} \ , \ \partial_\tau \phi = \partial_\sigma \tilde{\phi}$$

They are dual to each other on the mass-shell

Symmetrization:

$$L_{sim} = \frac{1}{2}\dot{\phi}\,\tilde{\phi}' + \frac{1}{2}\phi'\,\dot{\tilde{\phi}} - \frac{1}{2}\phi'^2 - \frac{1}{2}\tilde{\phi}'^2$$

invariant under

$$\phi \leftrightarrow \tilde{\phi}$$

equations of motion

$$\partial_\sigma \phi = \partial_\tau \tilde{\phi} \ , \ \partial_\tau \phi = \partial_\sigma \tilde{\phi}$$

proviso:

$$\int d\tau \left[\dot{\phi}\,\tilde{\phi}\right]\Big|_{\sigma=\sigma_0}^{\sigma=\sigma_1} = \int d\sigma \left[\phi'\,\tilde{\phi}\right]\Big|_{\tau=-\infty}^{\tau=\infty} = 0$$

(reproducing the previous conditions)

Furthermore L_{sim} can be diagonalized by introducing the chiral fields:

$$\phi_\pm = \frac{1}{\sqrt{2}}\left(\phi \pm \tilde{\phi}\right)$$

$$\phi = \frac{1}{\sqrt{2}}\left(\phi_+ + \phi_-\right)$$

$$\tilde{\phi} = \frac{1}{\sqrt{2}}\left(\phi_+ - \phi_-\right)$$

$$L_{sim}(\phi,\tilde{\phi}) = L_+(\phi_+) + L_-(\phi_-)$$

$$L_\pm(\phi_\pm) = \pm\frac{1}{2}\dot{\phi}_\pm\phi'_\pm - \frac{1}{2}\phi'^{\,2}_\pm$$

No manifest Lorentz invariance!

Only on shell ϕ_+ , ϕ_- become respectively functions of $\tau+\sigma , \tau-\sigma$

$$\dot{\phi}_+ = \phi'_+ \ (dual) \quad , \quad \dot{\phi}_- = -\phi'_- \ (anti-dual)$$

SYMMETRIES

L_{sim} , $L_\pm$ invariant under space-time translations

$L_\pm$ invariant under $\delta_L\phi_\pm = (\tau\pm\sigma)\phi'_\pm , \delta\phi_\pm = f(\tau\pm\sigma), \delta\phi_\pm = \tau\dot{\phi}_\pm + \sigma\phi_\pm'$

L_{sim} invariant under $\delta_L\phi = \tau\phi' + \sigma\tilde{\phi}' , \ \delta_L\tilde{\phi} = \tau\tilde{\phi}' + \sigma\phi'$

on-shell $\delta\phi = \tau\,\phi' + \sigma\,\dot{\phi}$

usual 2-dim Lorentz rotation

Lorentz invariance recovered on-shell!

The Lagrangians so far considered belong to a general class of first-order Lagrangians:

$$L = \frac{1}{2} q^i c_{ij} \dot{q}^j - V(q) \qquad i,j = 1,\ldots,N \qquad \det c_{ij} \neq 0$$

characterized by N primary constraints

$$T_j \equiv p_j - \frac{1}{2} q^i c_{ij} \approx 0$$

canonically conjugate momentum to q^j .

$$Q_{jk} \equiv \left\{ T_j, T_k \right\}_{PB} = c_{jk} \neq 0$$

$\boxed{\text{All constraints are second class}}$

Dirac bracket for any two functions of the phase-space variables:

$$\left\{ f, g \right\}_D \equiv \left\{ f, g \right\}_P - \left\{ f, T_j \right\}_P (Q^{-1})_{jk} \left\{ T_k, g \right\}_P$$

The Dirac formalism allows for a transition to the quantum theory:

$$i\{f,g\}_D \to [\hat{f},\hat{g}]$$

$$[q_i, q_j] = i(c^{-1})_{ij}$$

$$[q_i, p_j] = \frac{1}{2} i\delta_{ij}$$

$$[p_i, p_j] = -\frac{1}{4} i c_{ij}$$

$$L = \frac{1}{2} q^i c_{ij} \dot{q}^j - V(q) \quad i,j = 1,\ldots,N$$

$$L = \frac{1}{4}\int d\sigma\, d\sigma'\, \phi(\sigma)\,\varepsilon(\sigma-\sigma')\,\dot{\phi}(\sigma') - \frac{1}{2}\int d\sigma\, \dot{\phi}^2(\sigma)$$

$$[\phi(\sigma,\tau), \phi(\sigma',\tau)] = \pm i\varepsilon(\sigma-\sigma')$$

$$[\pi(\sigma,\tau), \phi(\sigma',\tau)] = \frac{i}{2}\delta(\sigma-\sigma')$$

$$[\pi(\sigma,\tau), \pi(\sigma',\tau)] = -\frac{1}{8} i\varepsilon(\sigma-\sigma')$$

showing that the field φ is non-local with local dynamics.

$\phi, \tilde{\phi}$ $[X, \tilde{X}]$ behave like “non-commuting” phase space type coordinates.

STRINGS IN TOROIDAL BACKGROUNDS

$$S = S_G + S_B = -\frac{T}{2}\int_0^{\pi} d\sigma \int_{-\infty}^{+\infty} d\tau \left[h^{\alpha\beta}\,\partial_\alpha X^i \partial_\beta X^j G_{ij} + \varepsilon^{\alpha\beta}\partial_\alpha X^i \partial_\beta X^j B_{ij} \right]$$

$$X^i = (X^a, X^\mu) \qquad i = 0,\ldots,D-1$$

periodic coordinates

$$X^a \approx X^a + 2\pi R w^a$$

The closed string background fields *G* and *B* are *DxD* matrices

$$G_{ij} = \begin{pmatrix} \hat{G}_{ab} & 0 \\ 0 & \eta_{\mu\nu} \end{pmatrix} \quad , \quad B_{ij} = \begin{pmatrix} \hat{B}_{ab} & 0 \\ 0 & 0 \end{pmatrix}$$

In this case the Hamiltonian reads as:

$$H = \frac{1}{2} Z^t M Z + N + \overline{N}$$

$$M(E) = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}$$

generalized metric 2DX2D

$$Z = \begin{pmatrix} w^i \\ p_i \end{pmatrix}$$

$$E_{ij} = G_{ij} + B_{ij}$$

background matrix

2D column vector consisting of integer winding and momentum quantum numbers.

H results to be invariant under the elements

$g \in O(D,D;R)$ $g = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ *a,b,c,d* are *DxD* matrices

generating on *E* the following transformation:

$$E' = g(E) = (aE + b)(cE + d)^{-1}$$

A particular element of *O(D,D;R)*: Inversion of the background matrix E

$$E = G + B \rightarrow E' = G' + B' = E^{-1}$$

analog in *D* dimensions to the circle duality $R \rightarrow \frac{1}{R}$

under which $2\pi\alpha' P_i = G_{ij}\dot{X}^j + B_{ij}X^{j\,\prime} \leftrightarrow X^{i\,\prime}$

GENERAL "SIGMA MODEL"

$$i,j = 1,\dots,D$$

$$S = \int d^2\sigma\, e\left[\phi + A_i^a \nabla_a X^i + D_{ij}^{ab} \nabla_a X^i \nabla_b X^j\right] \qquad \nabla_a = e_a^m \nabla_m$$

$\det e_a^m$

Includes all fields

$$g_{mn} = e_m^a \eta_{ab} e_m^b$$

2-bien

Usual manifestly invariant Lorentz-invariant sigma-model obtained if:

$$\phi = 0,\ A_i^a = 0\ ,\ D_{ij}^{ab} = \frac{1}{2}(\eta^{ab} G_{ij} + \varepsilon^{ab} B_{ij})$$

One can choose the "kinetic" term in the general form:

$$S = \frac{1}{2}\int d^2\sigma e\left[C_{ij}(X)\nabla_0 X^i \nabla_1 X^j + M_{ij}(X)\nabla_1 X^i \nabla_1 X^j\right]$$

symmetric

manifestly diffeomorphism and Weyl invariant but not Lorentz invariant

$$\delta e_m^a = \lambda e_m^a \longrightarrow \boxed{\text{Weyl invariance}}$$

$$e'^a_n(\sigma' = \sigma - \xi) = e_m^a \frac{\partial \sigma^m}{\partial \sigma'^n} \quad \rightarrow \quad \delta e_n^a = e_m^a \partial_n \xi^m + \partial_m e_n^a \xi^m$$

two-dimensional reparametrizations

$$e'^a_n = \Lambda^a_b e^b_n \rightarrow \delta e^a_n = \omega^a_b e^b_n \quad ; \quad \omega_{ab} = -\omega_{ba}$$

Lorentz transformations

The action variation of the action under these latter transformazion is proportional to the energy-momentum tensor

The vanishing of the energy-momentum tensor on the classical implies:

$$C = MC^{-1}M$$

while its vanishing at quantum level requires that *C* has zero signature

$$C = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad I = diag(1,\ldots,1) \longrightarrow \text{D-dimensional matrix}$$

The action describes a mixture of D left and D right scalars:

$$\mathrm{X}^i = (X_+^\mu\ , X_-^\mu) \qquad\qquad \mu = 1,\ldots,D$$

in terms of which one can define the non-chiral basis of fields:

$$X^\mu = \frac{1}{\sqrt{2}}(X_+^\mu + X_-^\mu) \quad , \quad \tilde{X}^\mu = \frac{1}{\sqrt{2}}(X_+^\mu - X_-^\mu)$$

Introducing the basis of non chiral fields, one has:

$$C = \Omega \quad ; \quad \Omega = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} = \Omega^{-1} \quad ; \quad \Omega = M\Omega M$$

with

$$M = \pm \begin{pmatrix} a & b^T \\ b & \tilde{a} \end{pmatrix} \quad a = a^T \qquad \tilde{a} = \tilde{a}^T$$

and

$$a = \tilde{G}^{-1}, \qquad \tilde{a} = G^{-1}, \qquad \tilde{G}^{-1} \equiv G - BG^{-1}B$$

$$b = BG^{-1}, \qquad B^T = -B, \qquad G^T = G$$

parametrized by D^2 elements of the symmetric matrix G and antisymmetric matrix B

Hence

$$S = \frac{1}{2}\int d^2\sigma e\left[C_{ij}(\mathrm{X})\nabla_0 \mathrm{X}^i \nabla_1 \mathrm{X}^j + M_{ij}(\mathrm{X})\nabla_1 \mathrm{X}^i \nabla_1 X^j\right]$$

with *C* and *M* dictated by the requirement of Lorentz-invariance is invariant under the combined *O(D,D)* transformations of X^i *and* the matrix of coupling parameters *M:*

$$\mathrm{X}' = \Lambda^{-1}\mathrm{X}, \quad M' = \Lambda^T M\Lambda, \quad \Lambda^T\Omega\Lambda = \Omega \qquad \Lambda \in O(D,D)$$

The Lorentz invariance constraint $\Omega = M\Omega M \Rightarrow M \in O(D,D)$

and its invariance under Λ.

S is invariant under the *duality transformations*:

$$\mathrm{X}' = \Omega\mathrm{X}, \qquad M' = \Omega M\Omega = M^{-1}$$

$$X \leftrightarrow \tilde{X}, \quad a \leftrightarrow \tilde{a}, \quad b \leftrightarrow b^T, \quad (G+B) \leftrightarrow (G+B)^{-1},$$

$$G \leftrightarrow \tilde{G}, \quad \tilde{G} = (G - BG^{-1}B)^{-1}, \qquad BG^{-1} \leftrightarrow -G^{-1}B$$

In component form one has:

$$S\left[X,\tilde{X}\right]=\frac{1}{2}\int d^2\sigma\, e(\nabla_0 X^\mu \nabla_1 \tilde{X}_\mu + \nabla_0 \tilde{X}^\mu \nabla_1 X_\mu - a_{\mu\nu}\nabla_1 X^\mu \nabla_1 X^\nu$$

$$-\bar{a}^{\mu\nu}\nabla_1\tilde{X}_\mu \nabla_1 \tilde{X}_\nu - 2b^\mu_\nu \nabla_1 X^\nu \nabla_1 \tilde{X}_\mu) \qquad \nabla_a = e^m_a \partial_m$$

$$a=\tilde{G}^{-1}, \quad \tilde{a}=G^{-1}, \quad \tilde{G}^{-1}=G-BG^{-1}B, \quad b=BG^{-1}$$

dictated by Lorentz invariance conditions

Integrating over $\tilde{X}^\mu$ (eliminating if from its equation of motion)

$$S[X]=\frac{1}{2}\int d^2\sigma\left(\sqrt{|h|}h^{ab}+\varepsilon^{ab}\right)\left(G+B\right)_{ij}\partial_a X^i \partial_b X^j$$

Integrating over X^μ

$$S\left[\tilde{X}\right]=\frac{1}{2}\int d^2\sigma(\sqrt{|h|}h^{ab}+\varepsilon^{ab})(G+B)^{-1\mu\nu}\partial_m \tilde{X}_\mu \partial_n \tilde{X}_\nu$$

- Work in progress: quantization of the manifestly duality simmetric action according to the Dirac quantization scheme already considered for the circle compactification.

- Geometrical interpretation of T-duality in the toroidal compactification

IMPLICATIONS ON THE GRAVITATIONAL EFFECTIVE THEORY

As in the standard manifestly Lorentz invariant formulation, there is a correspondence between the sigma model which describes a string in a background and the vertex operators associated with the physical states, now expressed in terms of X and its dual.

Comparing the spectrum of the duality symmetric theory with the standard formulation, one would expect that only the number of the zero-modes is doubled while the set of oscillators remains the same.

The states have to fulfill constraints coming from the request of local Lorentz invariance (satisfied on the equations of motion) plus "diffeomorphism" constraints.

The set of on-shell vertex operators creating the closed string physical states is given by the product of “left” and “right” vertex operators, each of them depending on the independent momenta

$$p_+, p_-$$

For a graviton one should have something like

$$V \approx \xi_{\mu\nu}(p_+, p_-) : \partial X^+ \bar{\partial} X^- e^{i(p_+ X^+ + ip_- X^-)} :$$

$$p_+^\mu \xi_{\mu\nu} = p_-^\mu \xi_{\mu\nu} = 0$$

3-graviton scattering amplitude

Effective action

$$S_3 = \int d^D X d^D \tilde{X} \left[R_3(\partial) - R_3(\bar{\partial}) \right]$$

$$\partial_\mu \approx \partial_{+\mu} + \partial_{-\mu} = \frac{\partial}{\partial X^\mu} \qquad \bar{\partial}_\mu \approx \bar{\partial}_{+\mu} - \bar{\partial}_{-\mu} = \frac{\partial}{\partial \tilde{X}^\mu}$$

3-graviton term in the expansion of the Ricci scalar for the metric

$$G_{\mu\nu}(X, \tilde{X}) = \eta_{\mu\nu} + h_{\mu\nu}(X, \tilde{X})$$

To establish the correspondence with the usual low-energy gravitational effective action

◆ assume that $G_{\mu\nu}$ does not depend on $\tilde{X}$

(considering the case when $\tilde{X}$ is compactified on a small torus of radius $\tilde{R} << \sqrt{\alpha'}$ so that the radius of the *X*-space is large) ;

◆ integrate over $\tilde{X}$ thus obtaining in this way a factor which rescales the usual low-energy coupling constants.

CONCLUSION

- T-duality symmetric formulation of bosonic closed string seems to be a very helpful tool for shedding light on string gravity and its corrections to the Einstein-Hilbert action.

- But it also gives the possibilities of studying more geometrical issues such as Complex Generalized Geometry and the role of Courant brackets in the Double Field Theory, analogue to the one of the Lie brackets in General Relativity.

Many things to understand….!

THANKS