Action for the eleven dimensional multiple M-wave system

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Outline

M-branes and D-branes and mDp

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- SYM description was the basis for the search for a more complete nonlinear description of mDp system: [Myers 1999] (purely bosonic), [Sorokin 03], [Howe, Linstrom, Wulff 2005-07] (boundary fermion SSP)....

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- The general eqs [IB 2010] were specialized for the case of 11D pp-wave superspace [IB 2011] and shown to reproduce (in some limit) the eqs of the BMN Matrix model [Berenstein, Maldacena and Nastase 2002].

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- The aim of this talk is to present such an action for mM0 in flat target superspace.

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- κ symmetry appears in its irreducible form in the so–called spinor moving frame formulation of superparticle [IB 1990, IB+AN 1996, IB 2007]

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Moving frame action and its twistor-like nature

- The spinor moving frame action of M0–brane reads

$$S_{M0} = \int_{W^1} \rho^{\#}\, \hat{E}^{=} = \int_{W^1} \rho^{\#}\, u_a^{=}\, E^a(\hat{Z})$$
$$= \frac{1}{16} \int_{W^1} \rho^{\#}\, (v_q{}^{-} \Gamma_a v_q{}^{-})\, \hat{E}^a ,$$

where $\rho^{\#}(\tau)$ is a Lagrange multiplier and $u_a^{=}$ is light–like $u^{=a} u_a^{=} = 0$.

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- These can be obtained from the ∞–reducible κ symm of S_{BS}, $\delta_\kappa \hat{\theta}^\alpha = p_a \tilde{\Gamma}^{a\alpha\beta} \kappa_\beta(\tau)$, by substituting for p_a the solution $p_a = \rho^{\#} u_a^{=}$ of the constraint $p_a p^a = 0$. Then $\epsilon^{+q} = 2\rho^{\#} v_q^{-\alpha} \kappa_\alpha$.

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- However, one might still find the origin of our $v_q^{-\alpha}$ a bit mysterious.

Moving frame and spinor moving frame

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- To clarify the nature of $v_q^{-\alpha}$, it is useful to consider the null–vector $u_a^{=}$ as an element of the *moving frame* matrix,

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- These Lorentz harmonics or Goldstone fields were used in superstring formulation of [IB & Zheltikhin, 1992] (see also [Gomis, Kamimura, West 2006]).
- For the case of superparticle the coset is $\frac{SO(1,D-1)}{[SO(1,1)\times SO(D-2)\ltimes K_{D-2}]}$ [Galperin, Howe, Stelle, 92, Galperin Delduc, Sokatchev 92, IB & Nurmagambetov 96]

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- $\Rightarrow du_a^{=}=-2u_a^{=}\Omega^{(0)}+u_a^i\Omega^{=i}$,
- $dv_q^{-\alpha}=-v_q^{-\alpha}\Omega^{(0)}+\frac{1}{4}\Omega^{ij}\gamma^{ij}_{qp}v_p^{-\alpha}-\frac{1}{2}\Omega^{=i}\gamma^i_{qp}v_p^{+\alpha}$, ...

On M0 equations as obtained from spinor moving frame action, and worldline geometry

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- Furthermore, $\hat{E}^a := \frac{1}{2}\, \hat{E}^\# u^{=a}$ suggests to consider $\hat{E}^\# = d\tau \hat{E}_\tau^\#$ as an einbein on W^1 (induced by the embedding).

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On M0 equations as obtained from spinor moving frame action, and worldline geometry

- Using moving frame we can split, in a Lorentz covariant manner, $\hat{E}^b \mapsto \hat{E}^b U_b^{(a)} = (\hat{E}^{=}, \hat{E}^{\#}, \hat{E}^i)$ (carrying SO(1,1) and SO(D-2) 'indices').
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Outline

mM0 action

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\mathcal{H} \; := \; & \mathcal{H}_{\#\#\#\#}(\mathbb{X}, \mathbb{P}, \Psi) = \frac{1}{2} tr \left(\mathbb{P}^i \mathbb{P}^i \right) + \mathcal{V}(\mathbb{X}) - 2 \, tr \left(\mathbb{X}^i \, \Psi \gamma^i \Psi \right) \; , \\
& \mathcal{V} := \mathcal{V}_{\#\#\#\#}(\mathbb{X}) = -\frac{1}{64} tr \left[\mathbb{X}^i, \mathbb{X}^j \right]^2 \equiv + \frac{1}{64} tr \left| \left[\mathbb{X}^i, \mathbb{X}^j \right] \right|^2 \; .
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- as for a single M0: $\hat{E}^{=} = \hat{E}^a u_a$, $\boxed{\hat{E}^{\#} = \hat{E}^a u_a, \quad \hat{E}^{+q} = d\hat{\theta}^\alpha v_\alpha^{+q}}$ *induced 1d SG*

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D\mathbb{X}^i \;&:=\; d\mathbb{X}^i + 2\Omega^{(0)}\mathbb{X}^i + \Omega^{ij}\mathbb{X}^j + [\mathbb{A},\mathbb{X}^i] , \\
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$\Uparrow$ induced 1d SG

Supersymmetry of the mM0 action

$$S_{mM0} = \int_{W^1} \rho^{\#}\, \hat{E}^{=} + \int_{W^1} (\rho^{\#})^3 \left(tr\left(-\mathbb{P}^i D\mathbb{X}^i + 4i\Psi_q D\Psi_q \right) + \hat{E}^{\#}\mathcal{H} \right) +$$

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- is invariant under the 16 parametric local worldline SUSY:

$$\begin{aligned} \delta_\epsilon \mathbb{X}^i &= 4i\epsilon^+\gamma^i\Psi \;, \quad \delta_\epsilon \mathbb{P}^i = [(\epsilon^+\gamma^{ij}\Psi), \mathbb{X}^j] \;, \\ \delta_\epsilon \Psi_q &= \frac{1}{2}(\epsilon^+\gamma^i)_q\mathbb{P}^i - \frac{i}{16}(\epsilon^+\gamma^{ij})_q[\mathbb{X}^i, \mathbb{X}^j] \;, \\ \delta_\epsilon A &= -\hat{E}^{\#}\epsilon^{+q}\Psi_q + (\hat{E}^+\gamma^i\epsilon^+)\mathbb{X}^i \;, \end{aligned}$$

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- The local SUSY acts on center of energy variables by a deformation of the irreducible κ-symm of the massless superparticle:

$$\delta_\kappa \hat{x}^a = -i\hat{\theta}\Gamma^a\delta_\kappa\hat{\theta} \ , \qquad \delta_\kappa\hat{\theta}^\alpha = \epsilon^{+q}(\tau) v_q^{-\alpha} \ , \qquad \delta_\kappa \rho^{\#} = 0 = \delta_\kappa u_a^{=} \ .$$

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- $\Rightarrow$ the center of energy motion of mM0 is *generically* not lightlike.
- It is characterized by an effective mass constructed from relative motion variables, $M^2 = M^2(\mathbb{X}^i,\mathbb{P}^i,\Psi_q)$.

On the center of energy motion

$$S_{mM0} = \int_{W^1} \rho^{\#} \hat{E}^{=} + \int_{W^1} (\rho^{\#})^3 \left(tr\left(-\mathbb{P}^i D\mathbb{X}^i + 4i\Psi_q D\Psi_q\right) + \hat{E}^{\#}\mathcal{H}\right) -$$

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$M^2 = 0$ as a BPS equation. Vanishing effective mass of all susy bosonic solutions

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- or, if not, can help to obtain a weak field approx. in $\mathbb{X}^i$ and Ψ_q (stopping the decomposition by hand at some power ≥ 4).

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- Although the search for generalizations for the case of mM2 is not promising to be easy in the light of recent results in [Gran, Greitz, Howe & Nilsson, 2012], neither it looks hopeless.
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- These enter covariant derivatives and the Chern-Simons term present in the BLG action, $\mathcal{L}_{CS} = \frac{1}{2}\int d^3y(ds^i \wedge A_i - \frac{1}{3}\epsilon_{ijk}s^i \wedge s^j \wedge s^k)$

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- The problem is that, although some κ–symmetric null-supermembrane actions are known, their κ–symmetry is of a different type then SUSY of the BLG model.

Thanks!

THANK YOU FOR YOUR ATTENTION!